Simplest Maths Booklet

Explaning and Exercises

Algebra and Geometry

Prep 2 - First Term 2025

Prepared by :
Mr.Mohamed El-Shourbagy

Simplest Maths

Maths-Prep2/first term (2025)

Unit 1

1. The cube root of a rational number.
2. The set of irrational numbers $\acute{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$ – Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree

Unit 2

1. Relation between two variables.
2. Slope of straight line.
3. Real life applications on the slope.

Unit 3

1. Collecting and organizing data.
2. The ascending and descending cumulative frequency tables and their graphical representation.
3. Mean.
4. Median.
5. Mode.

Mr.Mohamed El-Shourbagy / 01093149109

Explaining

Unit 1

Lesson (1)
The cube root of a Rational Number

The cube root of a rational number

Definition

The cube root of the rational number "a" is the number whose cube equal to a
The cube root of the rational number "a" is denoted by $\sqrt[3]{a}$

For example :

- $\sqrt[3]{8} = 2$ because $(2)^3 = 8$
- $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$

Notice that : $\sqrt[3]{0} = 0$

i.e. • The cube root of any number has the same sign of this number.

Remarks

1. $\sqrt[3]{a^3} = a$ For example : $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$
2. $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ For example : $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$

• You can use factorization to find the cube root of a perfect cube number , as in the following example.

Example 1 Find each of the following :

1. $\sqrt[3]{216}$
2. $\sqrt[3]{\frac{-8}{125}}$
3. $\sqrt[3]{0.064}$

Solution

1. $\sqrt[3]{216} = 2 \times 3 = 6$

216	2
108	2
54	2
27	3
9	3
3	3
1	

2 (2 2 2) ; 3 (3 3 3)

2. $\sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

8	2
4	2
2	2
1	

2

125	5
25	5
5	5
1	

5

3. $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{2 \times 2}{2 \times 5}$

$= \frac{4}{10} = 0.4$

64	2
32	2
16	2
8	2
4	2
2	2
1	

2 ; 2

1000	2
500	2
250	2
125	5
25	5
5	5
1	

2 ; 5

Remark

If "a" is a perfect cube number ,
then the equation : $x^3 = a$ has a unique solution in $\mathbb{Q}$, which is $\sqrt[3]{a}$

For example: • The equation : $x^3 = 8$ has a unique solution in $\mathbb{Q}$ which is $\sqrt[3]{8} = 2$

• The equation : $x^3 = 9$ has no solution in $\mathbb{Q}$ because 9 is not a perfect cube.

Applications

Remember that :

Knowing that : The volume of the sphere = $\frac{4}{3}\pi r^3$

- The volume of a cube = the edge length × itself × itself
- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6

For example: If the volume of a cube is 8 cm^3 , then :

- The edge length = $\sqrt[3]{8} = 2$ cm.
- The area of one face = $2 \times 2 = 4$ cm^2
- The lateral area = $4 \times 4 = 16$ cm^2
- The total area = $4 \times 6 = 24$ cm^2

[A] Find the value of X in each of the following :

1) $\sqrt[3]{X} = 5$	2) $\sqrt[3]{X} = -\frac{1}{4}$	3) $\sqrt[3]{X} = -\sqrt{4}$	4) $\sqrt[3]{X} - 3 = 1$
$X = 5^3$ $X = 125$	$X = (-\frac{1}{4})^3$ $X = -\frac{1}{64}$	$\sqrt[3]{X} = -2$ $X = (-2)^3$ $X = -8$	$\sqrt[3]{X} = 3 + 1$ $\sqrt[3]{X} = 4$ $X = 4^3$ $X = 64$

[B] Find the S.S of each of the following equations in Q :

1) $X^3 = 64$	2) $X^3 + 5 = 32$	3) $\frac{1}{5}X^3 = -200$	4) $(3X + 1)^3 = -8$
$X = \sqrt[3]{64}$ $X = 4$ S.S = { 4 }	$X^3 = 32 - 5$ $X^3 = 27$ $X = \sqrt[3]{27}$ $X = 3$ S.S = { 3 }	$X^3 = -200 \div \frac{1}{5}$ $X = -1000$ $X = \sqrt[3]{1000}$ $X = 10$ S.S = { 10 }	$(3X + 1) = \sqrt[3]{-8}$ $3X+1 = -2$ $X = \frac{-2-1}{3}$ $X = -1$ S.S = { –1 }

5) $2X^3 - 5 = X^3 + 3$	6) $(X + 3)^3 = 343$	7) $(5X - 2)^3 + 10 = 18$
$2X^3 - X^3 = 5 + 3$ $X^3 = 8$ $X = \sqrt[3]{8}$ $X = 2$ S.S = { 2 }	$X + 3 = \sqrt[3]{343}$ $X + 3 = 7$ $X = 7 - 3$ $X = 4$ S.S = { 4 }	$(5X - 2)^3 = 18 - 10$ $(5X - 2)^3 = 8$ $5X - 2 = \sqrt[3]{8} = 2$ $X = \frac{2+2}{5} = \frac{4}{5}$ S.S = { $\frac{4}{5}$ }

EX. (1): Complete the following:

Exercises

1	$\sqrt[3]{a^3} = \dots\dots\dots$
2	$\sqrt[3]{-8} = \dots\dots\dots$
3	$\lvert\sqrt[3]{-125}\rvert = \dots\dots\dots$
4	$\lvert\sqrt[3]{-125}\rvert = \sqrt{\dots\dots\dots}$
5	$\sqrt[3]{27} - \sqrt[3]{-27} = \dots\dots\dots$
6	$-\sqrt[3]{-1} - \sqrt{1} = \dots\dots\dots$
7	$\sqrt[3]{64 + \dots\dots\dots} = 5$
8	$\sqrt[3]{\dots\dots\dots} = 4$
9	$\sqrt{16} = \sqrt[3]{\dots\dots\dots}$
10	$\sqrt[3]{64} = \sqrt{\dots\dots\dots}$
11	If : $\sqrt[3]{64} = \sqrt{x}$, then $2x = \dots\dots\dots$
12	If : $x^2 = 5$, then $(x + \sqrt{5})^2 = \dots\dots\dots$ or $\dots\dots\dots$
13	$\frac{x^3}{y^3} = \frac{1}{64}$, then $\left(\frac{y}{x}\right)^2 = \dots\dots\dots$
14	If $8 = \sqrt[3]{x}$, then $x = \dots\dots\dots$
15	If $\sqrt[3]{x} = -\sqrt{4}$, then $x = \dots\dots\dots$

16	If $x^2 - y^2 = 60$ and $x + y = 5$, then $x - y =$ ……………
17	The solution set for the equation : $x^2 + 1 = 0$ in $\mathbb{Q}$ is ……………
18	The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{Q}$ is ……….
19	The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{Q}$ is ……………
20	The S.S. of the equation : $x^2 + 25 = 0$ in $\mathbb{Q}$ is ……………
21	The solution set of the equation : $(x^2 + 3)(x^2 + 1) = 0$ where $x \in \mathbb{Q}$ is ……………
22	The S.S. of the equation : $(x^2 - 1)(x + 5) = 0$ in $\mathbb{Q}$ is ……………
23	The S.S. of the equation : $(x^2 + 1)(x - 5) = 0$ in $\mathbb{Q}$ is ……………
24	The S.S. of the equation : $x^3 + 1 = 2$ in $\mathbb{Q}$ is ……………
25	The S.S. of the equation : $x(x^3 - 1) = 0$ in $\mathbb{Q}$ is ……………
26	The S.S. of the equation : $(x^2 + 3)(x^3 + 1) = 0$ is ……………

EX. (2): Choose the correct answer:

1	$\left(2\sqrt[3]{2}\right)^3 = \dots\dots$ (a) 4 (b) 8 (c) 16 (d) 40
2	📖 $\sqrt[3]{(-8)^2} = \dots\dots$ (a) 2 (b) –2 (c) 4 (d) – 4
3	$\sqrt{8} - \sqrt{2} = \dots\dots$ (a) $\sqrt{6}$ (b) 2 (c) $\sqrt{2}$ (d) 1
4	$\sqrt{25} - \sqrt[3]{-125} = \dots\dots$ (a) 10 (b) zero (c) 5 (d) ± 5
5	$-2\sqrt{3} \times \sqrt{3} = \dots\dots$ (a) $-2\sqrt{3}$ (b) -6 (c) $2\sqrt{3}$ (d) 6
6	$\sqrt{3}\left(\sqrt{11} + \sqrt{3}\right) = \dots\dots$ (a) $3\sqrt{11} + 2$ (b) $\sqrt{33} + 3$ (c) $11\sqrt{3} + 2$ (d) $2\sqrt{11} + 3$
7	$\sqrt{9} + \sqrt[3]{-27} = \dots\dots$ (a) 0 (b) -6 (c) -9 (d) ± 6
8	$\sqrt[3]{-8} + \sqrt{4} = \dots\dots$ (a) 4 (b) -4 (c) zero (d) 8
9	$\sqrt{25} = \sqrt[3]{\dots\dots}$ (a) 5 (b) 15 (c) 125 (d) -5

10	If : $\sqrt[3]{y} = -\sqrt{9}$, then y = ... (a) 3 (b) – 3 (c) – 27 (d) 27
11	$\sqrt{25} + \sqrt[3]{-27} = \sqrt{\dots\dots\dots\dots}$ (a) 8 (b) 4 (c) 2 (d) 5
12	$\sqrt[3]{27} = \sqrt{x+3}$, then x = ... (a) 3 (b) 6 (c) 9 (d) 12
13	If $x^3 = 64$, then $\sqrt{x}$ = ... (a) 4 (b) – 4 (c) 2 (d) – 2
14	The solution set for the equation : $x^2 = 2$ in $\mathbb{Q}$ is .... (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\varnothing$
15	The S.S. of the equation : $x^2 + 3 = 0$ in $\mathbb{Q}$ is .... (a) $\varnothing$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $\pm\sqrt{3}$
16	The S.S. of the equation : $x^2 + 5 = 9$ where $x \in \mathbb{Q}$ is .... (a) $\{4\}$ (b) $\{-2, 2\}$ (c) $\varnothing$ (d) $\{13\}$
17	The S.S. of the equation : $x^3 + 8 = 0$ in $\mathbb{Q}$ is .... (a) $\{2\}$ (b) $\{2\sqrt{2}\}$ (c) $\{-2\}$ (d) $\{2, -2\}$
18	The solution set for the equation : $x^3 + 9 = 8$ in $\mathbb{Q}$ is .... (a) $\{8\}$ (b) $\{9\}$ (c) $\{3\}$ (d) $\{-1\}$
19	The S.S. of the equation : $x^3 + 27 = 0$ in $\mathbb{Q}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3\sqrt{3}\}$ (d) $\{\pm 3\sqrt{3}\}$
20	The S.S. in $\mathbb{R}$ of the equation : $x^3 + 11 = 12$ in $\mathbb{Q}$ is .... (a) $\{11\}$ (b) $\{12\}$ (c) $\{1\}$ (d) $\{3\}$

EX. (3): Answer the following:

1	**Find the value of x in each of the following :** 📖 $\sqrt[3]{x} = 5$
2	**Find the value of x in each of the following :** 📖 $x^3 = -8$
3	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 📖 $x^3 + 27 = 0$
4	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 📖 $8x^3 + 7 = 8$
5	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 📖 $(x + 3)^3 = 343$
6	**Find the S.S. of each of the following equations in $\mathbb{Q}$:** 📖 $(5x - 2)^3 + 10 = 18$
7	Find the edge length of a cube with volume = $15\frac{5}{8}$ cm^3.
8	📖 Find the inner edge length of a cube vessel with capacity of one litre.
9	📖 Find the diameter length of a sphere whose volume = $\frac{1372}{81}\pi$ cube unit.

Explaining

Lesson (2)

The Set of Irrational Numbers

1 The square root of the perfect square rational number is a rational number.

For example :

$\sqrt{1}$, $\sqrt{\frac{1}{9}}$, $\sqrt{\frac{25}{4}}$, $\sqrt{0.09}$, ... that are rational numbers.

- But the square root of the rational number which is not a perfect square is not a rational number.

For example :

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{\frac{2}{5}}$ ... they are not rational numbers

Because there is no rational number whose square is : 2 or 3 or $\frac{2}{5}$ or ...

2 The cube root of the perfect cube rational number is a rational number.

For example :

$\sqrt[3]{8}$, $\sqrt[3]{-64}$, $\sqrt[3]{0.027}$, .. they are rational numbers.

- But the cube root of the rational number which is not a perfect cube is not a rational number.

For example :

$\sqrt[3]{2}$, $\sqrt[3]{4}$, $\sqrt[3]{\frac{5}{8}}$, ... they are not rational numbers.

Because there is no rational number whose cube equals 2 or 4 or $\frac{5}{8}$ or ....

3 The numbers $\frac{22}{7}$, 3.14 , 3.142 , ... are rational numbers , each of them represents an approximating value of the number π

But the number π which denotes the ratio between the circumference of the circle and its diameter length is not a rational number.

From the previous, we deduce that :

There is another set of numbers which are not rational numbers. This set is called "the set of irrational numbers" and it is denoted by $\grave{\mathbb{Q}}$

Notice that :

$\mathbb{Q}$ and $\grave{\mathbb{Q}}$ are disjoint sets. **i.e.** $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

i.e. The irrational number is represented by an infinite decimal and not recurring.

Remark

- $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$, where $a \geq 0$ For example : $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$
- $(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$, where $a \in \mathbb{Q}$ For example : $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

Remark

From the previous , we can deduce that :

Each irrational number lies between two rational numbers.

Example (3) **Prove that :**

1 $\sqrt{3}$ lies between 1.7 and 1.8 **2** $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution

1 $\because (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$

$\because 2.89 < 3 < 3.24$ $\therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\therefore 1.7 < \sqrt{3} < 1.8$

i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows :

$\because \sqrt{3} \simeq 1.73$ $\because 1.7 < 1.73 < 1.8$

$\therefore 1.7 < \sqrt{3} < 1.8$ $\therefore \sqrt{3}$ lies between 1.7 and 1.8

2 $\because (\sqrt[3]{12})^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

$\because 10.648 < 12 < 12.167$ $\therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows :

$\sqrt[3]{12} \simeq 2.289$ $\because 2.2 < 2.289 < 2.3$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$ $\therefore \sqrt[3]{12}$ lies between 2.2 and 2.3

Representing an irrational number on the number line

Therefore we can deduce that :

Each irrational number can be represented by a point on the number line.

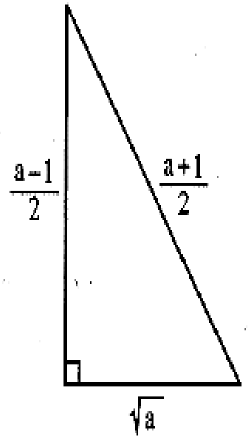
Generally

To draw a line segment with length $\sqrt{a}$ length unit where $a > 1$,

draw a right-angled triangle in which

the length of one side of the right-angle = $\frac{a-1}{2}$ length unit

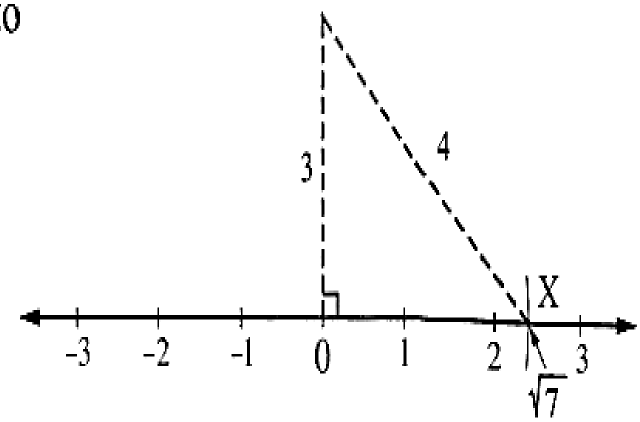
and the length of the hypotenuse = $\frac{a+1}{2}$ length unit.



Example (4)

Draw a line segment with length = $\sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :

Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$



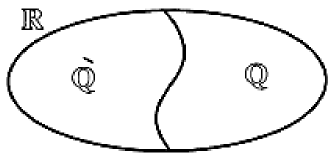
Explaining

Lesson (3)
The Set of Real Numbers

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

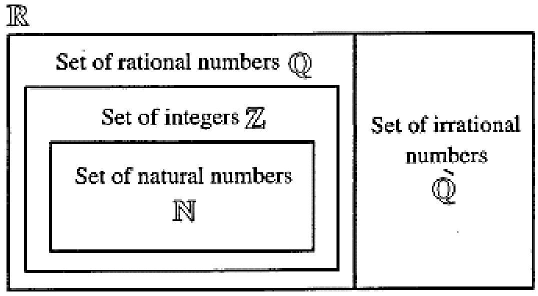


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

- **The opposite Venn diagram shows that :**

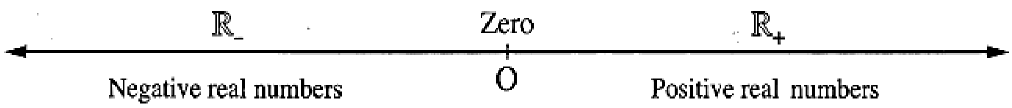
  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

  and $\grave{\mathbb{Q}} \subset \mathbb{R}$



$\mathbb{R}_- = \{x : x \in \mathbb{R} , x < \text{zero}\}$

$\mathbb{R}_+ = \{x : x \in \mathbb{R} , x > \text{zero}\}$



Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R} , x \geq 0\}$ and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R} , x \leq 0\}$ and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non-zero real numbers) is denoted by $\mathbb{R}^*$

  i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

EX. (1): Complete the following:

Exercises

1	If : $x < -\sqrt{7} < x+1$, then $x =$	(where x is an integer)
2	If : $x < \sqrt{15} < x+1$, $x \in \mathbb{Z}$, then $x =$	
3	If : $x < \sqrt{19} < x+1$, then $x =$	
4	If $x < \sqrt{20} < x+1$, $x \in \mathbb{Z}$, then $x =$	
5	If $x < \sqrt{10} < x+1$, $x \in \mathbb{Z}_+$, then $x =$	
6	$\mathbb{Q} \cap \mathbb{Q}' =$	$\mathbb{R} - \mathbb{Q}' =$ ....
7	$\mathbb{Q} \cup \mathbb{Q}' =$	$\mathbb{R} - \mathbb{Q} =$ ....
8	$\mathbb{R}_+ \cup \mathbb{R}_- =$	$\mathbb{R}_+ \cap \mathbb{R}_- =$ ....
9	The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is	
10	The multiplicative inverse of the number : $\left(\sqrt{3}+\sqrt{2}\right)$ is	

EX. (2): Choose the correct answer:

1	If : $\frac{3}{a+2}$ is a rational number then $a \neq$ .... (a) 3 (b) 5 (c) – 2 (d) zero
2	If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then n = ... (a) 25 (b) 5 (c) –5 (d) 24
3	The irrational number in the following numbers is .... (a) $\sqrt{\frac{1}{9}}$ (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{3}$ (d) $\sqrt[3]{27}$
4	The irrational number in the following numbers is .... (a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$
5	The irrational number lies between 2 and 3 is .... (a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$
6	The irrational number lies between 3 and 4 is .... (a) 3.5 (b) $\frac{1}{8}$ (c) $\sqrt{20}$ (d) $\sqrt{13}$
7	🕮 The area of a square whose side length is $\sqrt{3}$ cm. = ... cm^2 (a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6
8	🕮 The square whose area is 10 cm^2, its side length is .... cm. (a) 5 (b) – 5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$
9	The multiplicative inverse of $\frac{\sqrt{3}}{3}$ is .... (a) $\sqrt{3}$ (b) 1 (c) 3 (d) $-\sqrt{3}$

10	The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is .... (a) $-\sqrt{10}$ (b) $\sqrt{5}$ (c) $-2\sqrt{5}$ (d) $2\sqrt{5}$
11	The multiplicative inverse of the number $\sqrt{5}$ is .... (a) $-\sqrt{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $5\sqrt{5}$ (d) $\frac{5}{\sqrt{5}}$
12	$\mathbb{Q} \cap \grave{\mathbb{Q}}$ = ... (a) $\{0\}$ (b) $\varnothing$ (c) $\mathbb{R}$ (d) $\mathbb{Q}$
13	If : $\sqrt[3]{y} = -\sqrt{9}$, then y = ... (a) 3 (b) – 3 (c) – 27 (d) 27
14	$\sqrt{25} + \sqrt[3]{-27} = \sqrt{\ldots\ldots\ldots\ldots}$ (a) 8 (b) 4 (c) 2 (d) 5
15	$\sqrt[3]{27} = \sqrt{x+3}$, then x = ... (a) 3 (b) 6 (c) 9 (d) 12
16	The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is .... (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$
17	$\sqrt[3]{27} = \sqrt{x+3}$, then x = ... (a) 3 (b) 6 (c) 9 (d) 12
18	If $x^3 = 64$, then $\sqrt{x}$ = ... (a) 4 (b) – 4 (c) 2 (d) – 2
19	The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is .... (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$

EX. (3): Answer the following:

1	**Find an approximated value for each of the following numbers :** $\sqrt{11}$ "to the nearest hundredth".
2	**Find an approximated value for each of the following numbers :** $\sqrt[3]{-9}$ "to the nearest tenth".
3	**If x is an integer , find the value of x in each of the following cases :** $x < \sqrt{2} < x + 1$
4	**If x is an integer , find the value of x in each of the following cases :** 🕮 $x < \sqrt{80} < x + 1$
5	**Prove that :** $\sqrt{2}$ is included between 1.4 and 1.5
6	**Prove that :** 🕮 $\sqrt[3]{15}$ is included between 2.4 and 2.5
7	**Determine the point that represents each of the following numbers on the number line :** (1) $\sqrt{3}$ (2) $-\sqrt{11}$ (3) $\sqrt{10}$

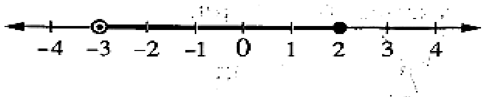
8	Find the side length of a square whose area is 5 cm^2. Is the side length a rational number ?
9	Find the edge length of a cube whose volume is 1.728 cm^3. Is the edge length a rational number ?
10	**Arrange the following numbers ascendingly :** 1 $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$ 2 $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$
11	**Put the suitable sign (> , < or =) :** 1 $\sqrt{5}$ .... 2 2 $\sqrt{7}$ .... 2.6 3 $\sqrt[3]{24}$ .... 3 4 $\sqrt[3]{-24}$ .... -2 5 $3-\sqrt{5}$ .... $\sqrt[3]{-1}$ 6 $\sqrt[3]{8}$ .... $\sqrt{4}$
12	A cube whose total area is 13.5 cm^2. Find its edge length. Is the edge length a rational number ?
13	**Arrange the following numbers descendingly :** 1 $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$ 2 $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$
14	**Find the value of x in each of the following cases and determine whether $x \in \mathbb{Q}$ or $x \in \mathbb{Q}'$:** 1 $5x^2 = 10$ 2 $4x^2 = 9$ 3 $x^3 = 125$ 4 $(x-1)^2 = 4$

Explaining

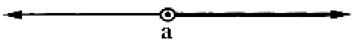
Lesson (4)

The Intervals

- It is represented on the number line as in the figure :



Notice that : $-3 \notin]-3 , 2]$, $2 \in]-3 , 2]$

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \leq x \leq b\}$		• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$		• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \leq x < b\}$		• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \leq b\}$		• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \geq a\}$		$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$		$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \leq a\}$		$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$		$a \notin]-\infty , a[$

Remarks

1 $\mathbb{R} =]-\infty , \infty[$

2 $\mathbb{R}_+ =]0 , \infty[$

3 $\mathbb{R}_- =]-\infty , 0[$

4 The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$

5 The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

Operations on intervals

You studied before the sets and how to carry out the operations of intersection , union , difference and complement on them.

For example:

If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5 , 6\}$, then :

- $X \cap Y$ = the set of elements which are common in X and Y $= \{3 , 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating $= \{1 , 2 , 3 , 4 , 5 , 6\}$
- $X - Y$ = the set of elements which are in X and not in Y $= \{1 , 2\}$
- $Y - X$ = the set of elements which are in Y and not in X $= \{5 , 6\}$
- If the universal set $U = \{1 , 2 , 3 , 4 , 5 , 6 , 7\}$,

  then the complement of X which is denoted by $\grave{X} = U - X$

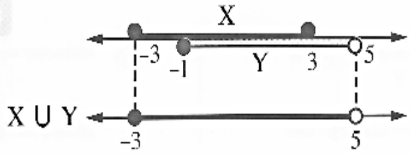
  i.e. $\grave{X}$ = the set of elements which are in U and not in X $= \{5 , 6 , 7\}$

Example 1 If $X = [-3 , 3]$ and $Y = [-1 , 5[$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$

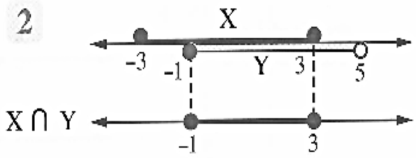
Solution

1.



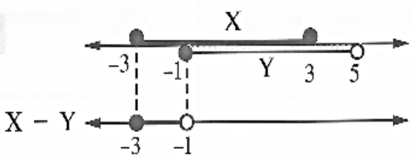
$X \cup Y = [-3 , 5[$

2.



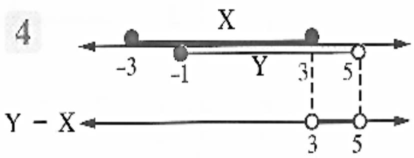
$X \cap Y = [-1 , 3]$

3.



$X - Y = [-3 , -1[$

4.



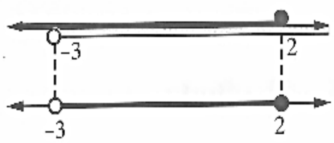
$Y - X =]3 , 5[$

Example 2 Find each of the following :

1. $]-\infty , 2] \cap]-3 , \infty[$
2. $]-\infty , 3] \cup [-2 , 5[$
3. $[5 , \infty[-]5 , \infty[$
4. $[2 , \infty[\cap]-\infty , 2[$

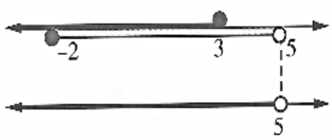
Solution

1.



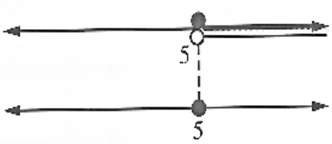
$]-\infty , 2] \cap]-3 , \infty[=]-3 , 2]$

2.



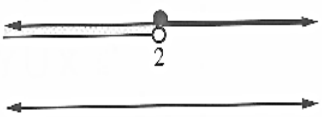
$]-\infty , 3] \cup [-2 , 5[=]-\infty , 5[$

3.



$[5 , \infty[-]5 , \infty[= \{5\}$

4.



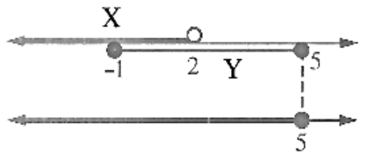
$[2 , \infty[\cap]-\infty , 2[= \varnothing$

Example 3

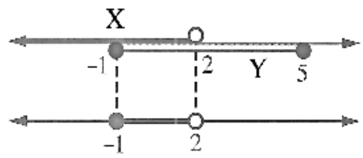
If $X =]-\infty , 2[$ and $Y = [-1 , 5]$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$
5. X'
6. Y'

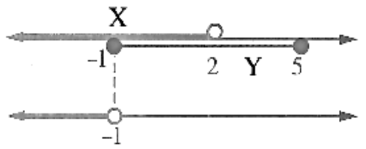
Solution

1.


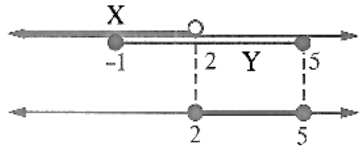
$X \cup Y =]-\infty , 5]$

2.


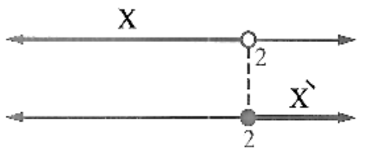
$X \cap Y = [-1 , 2[$

3.


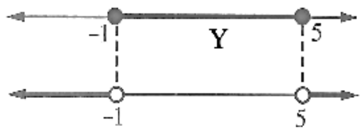
$X - Y =]-\infty , -1[$

4.


$Y - X = [2 , 5]$

5.


$X' = [2 , \infty[$

6.


$Y' =]-\infty , -1[\cup]5 , \infty[$

$= \mathbb{R} - [-1 , 5]$

Example 4

If $X = [1 , 4[$ and $Y = \{1 , 4\}$, find :

1. $X \cap Y$
2. $X \cup Y$
3. $X - Y$
4. $Y - X$

Solution

1. $X \cap Y = \{1\}$
2. $X \cup Y = [1 , 4]$
3. $X - Y =]1 , 4[$
4. $Y - X = \{4\}$

EX. (1): Complete the following:

Exercises

1	$[1 , 3] \cup [2 , 5[=$ ....
2	$]1 , 3] \cup [2 , 5] =$ ....
3	$]-\infty , 1] \cup [-4 , \infty[=$ ....
4	$]3 , 5[\cup \{3 , 5\} =$ ....
5	$]-2 , 2] \cup \{-2 , 0\} =$ ....
6	$]5 , 7[\cup \{5 , 7\} =$ ....
7	$\mathbb{N} \cap]1 , 2[=$ ....
8	$]1 , 7[\cap]3 , 5[=$ ....
9	$[-3 , 1[\cap [-1 , 4[=$ ....
10	$[-2 , 5] \cap]4 , 6] =$ ....
11	$]-3 , 5] \cap [0 , 3[=$ ....
12	$[1 , 5] - \{1 , 5\} =$ ....
13	$[2 , 5] - \{5\} =$ ....
14	$[2 , 5] - \{2 , 5\} =$ ....

15	$[3 , 4] - \{3 , 4\} =$ ....
16	$[3 , 5] - \{3\} =$ ....
17	$[3 , 7[-]-2 , 5] =$ ....
18	$[-4 , 6] -]-4 , 6[=$ ....
19	$[-1 , 5] -]-1 , 5[=$ ....
20	$[2 , 7] -]2 , 7[=$ ....

EX. (2): Choose the correct answer:

1 $\sqrt[3]{8}$ .... $]-\infty, 4[$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 $5 \in$ ....

(a) $]5, \infty[$ (b) $]-\infty, 5[$ (c) $(3, 5)$ (d) $[-5, \infty[$

3 The opposite figure represents the interval ....

(a) $[-4, 8[$ (b) $[8, -4]$ (c) $[-4, 8]$ (d) $]-4, 8[$

4 $\mathbb{R} =$ ....

(a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c) $]-\infty, \infty[$ (d) $\mathbb{Q} \cap \grave{\mathbb{Q}}$

5 $\mathbb{R}_+ =$ ....

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $[0, \infty[$ (d) $]-\infty, 0]$

6 $\mathbb{R}_- =$ ....

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $[0, \infty[$ (d) $]-\infty, 0]$

7 The set of non-negative real numbers = ...

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $[0, \infty[$ (d) $]-\infty, 0]$

8 The set of non-positive real numbers = ...

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $[0, \infty[$ (d) $]-\infty, 0]$

9 $]-1, 3] \cap [-3, -1] =$ ....

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

10	$[1 , 5] \cap]-2 , 3] =$ .... (a) $\{1 , 3\}$ (b) $]1 , 3[$ (c) $[1 , 3]$ (d) $[1 , 3[$
11	$]-3 , 5[\cap [0 , 3[=$ .... (a) $[0 , 3]$ (b) $[0 , 3[$ (c) $]-3 , 0[$ (d) $[3 , 5[$
12	$[2 , 7] - \{ 2 , 7 \} =$ .... (a) $[1 , 6]$ (b) $\varnothing$ (c) $]2 , 7[$ (d) $\{ 0 \}$
13	$[-2 , 5] - \{-2 , 6\} =$ .... (a) $]-2 , 5[$ (b) $]-2 , 6[$ (c) $]-2 , 5]$ (d) $[-2 , 5[$
14	$[-3 , 7] - \{-3 , 7\} =$ .... (a) $[-3 , 7[$ (b) $]-3 , 7]$ (c) $]-3 , 7[$ (d) $(0 , 0)$

EX. (3): Answer the following:

1	If $X = [-1, 4]$, $Y = [3, \infty[$, $Z = \{3, 4\}$, **find using the number line** (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Z$
2	If $X = [-2, 1]$ and $Y = [0, \infty[$ **Find :** (1) $X \cap Y$ (2) $X \cup Y$
3	If $X = [3, \infty[$, $Y =]-4, 8[$ **Find :** (1) $X \cup Y$ (2) $X \cap Y$ (3) $\grave{X}$
4	If $X = [-1, 4]$ and $Y = [2, 7]$, **then find each of :** (1) $X \cap Y$ (2) $Y \cup X$
5	If $X = [-2, 1]$, $Y = [0, \infty[$ **Find :** (1) $X \cap Y$ (2) $X \cup Y$ (3) $Y - X$
6	If $X = [-1, 4]$, $Y = [3, \infty[$, **find using the number line each of :** (1) $X \cup Y$ (2) $X - Y$
7	**Find each of the following :** (1) $[0, 5] \cup [3, 8[$ (2) $[1, 5] \cap]-2, 3]$
8	If $X = [-2, 3]$, $Y = [1, 5[$, **then find by using the number line :** $X \cup Y$, $X - Y$
9	If $X = [-2, 4]$ and $Y =]2, \infty[$, **find each of the following using the number line :** (1) $X \cap Y$ (2) $X - Y$
10	If $X = [-1, 4]$, $Y = [2, 7]$, then find each of the following by using the number line : (1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Y$
11	If $X = [1, 5]$, $Y = [2, 7]$ **Find by using the number line :** (1) $X \cap Y$ (2) $X \cup Y$

Explaining

Lesson (5)
Operations on the Real Numbers

Properties of addition of real numbers

The additive neutral :

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example : $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + (-\sqrt[3]{5}) = -\sqrt[3]{5}$

The additive inverse of every real number :

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example :

- The additive inverse of the number $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + (-\sqrt{3}) = 0$
- The additive inverse of the number $2 + \sqrt{5}$ is $-(2 + \sqrt{5})$ and equals $-2 - \sqrt{5}$
- The additive inverse of the number $3 - \sqrt{2}$ is $-(3 - \sqrt{2})$ and equals $\sqrt{2} - 3$
- The additive inverse of the number zero is itself.

The Properties of multiplication operation of real numbers

The multiplicative neutral :

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example :

- $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number :

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

For example :

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$

Notice that :

Both the number and its multiplicative inverse have the same sign.

- The multiplicative inverse of the number 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

There is no multiplicative inverse for the number zero because $\frac{1}{\text{zero}}$ is meaningless (**i.e.** undefined)

Remark

- Since each non-zero real number has a multiplicative inverse then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

  i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

Then we can deduce that :

The division operation in $\mathbb{R}$ is not commutative and it is not associative.

Lesson (6): Operations on the Square roots

Explaining

Remarks

1 $\sqrt{a^2+b^2} \neq a+b$, $\sqrt{a^2-b^2} \neq a-b$

For example :

- $\sqrt{6^2+8^2} \neq 6+8$ because

  $\sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$

- $\sqrt{25-9} \neq 5-3$ because $\sqrt{25-9} = \sqrt{16} = 4$

2 $a\sqrt{b} = \sqrt{a^2 b}$

For example :

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

3 $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ where $b \neq 0$

This operation is carried out to make the denominator an integer.

Notice that :

$(a+b)(a-b) = a^2 - b^2$

Notice that :

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$

For example :

- $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$
- $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

Example 1 Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1. $\sqrt{27}$
2. $5\sqrt{54}$
3. $3\sqrt{\frac{2}{3}}$
4. $\frac{\sqrt{84}}{\sqrt{7}}$

Solution

1. $\sqrt{27} = \sqrt{9 \times 3}$
$= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

2. $5\sqrt{54} = 5\sqrt{9 \times 6} = 5 \times \sqrt{9} \times \sqrt{6}$
$= 5 \times 3 \times \sqrt{6} = 15\sqrt{6}$

3. $3\sqrt{\frac{2}{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3 \times \frac{\sqrt{6}}{3} = \sqrt{6}$

Another solution :

$3\sqrt{\frac{2}{3}} = \sqrt{3^2 \times \frac{2}{3}} = \sqrt{3 \times 2} = \sqrt{6}$

4. $\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Example 2 Simplify to the simplest form :

1. $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5}$
2. $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}}$
3. $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}}$

Solution

1. $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5} = \sqrt{9 \times 5} - 2\sqrt{4 \times 5} + 2\sqrt{5}$
$= \sqrt{9} \times \sqrt{5} - 2 \times \sqrt{4} \times \sqrt{5} + 2\sqrt{5}$
$= 3\sqrt{5} - 2 \times 2\sqrt{5} + 2\sqrt{5}$
$= 3\sqrt{5} - 4\sqrt{5} + 2\sqrt{5} = \sqrt{5}$

2. $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}} = 2\sqrt{9 \times 2} + \sqrt{25 \times 2} - 42 \times \frac{\sqrt{1}}{\sqrt{2}}$
$= 2 \times \sqrt{9} \times \sqrt{2} + \sqrt{25} \times \sqrt{2} - 42 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= 2 \times 3\sqrt{2} + 5\sqrt{2} - 21\sqrt{2} = -10\sqrt{2}$

3. $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}} = 2\sqrt{9 \times 3} - 3 \times \frac{\sqrt{1}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= 6\sqrt{3} - \sqrt{3} - \frac{6\sqrt{3}}{3}$
$= 6\sqrt{3} - \sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$

EX. (1): Complete the following:

Exercises

1	The multiplicative neutral in $\mathbb{R}$ is ………. and the additive neutral in $\mathbb{R}$ is ……….
2	📖 The additive inverse of the number $1-\sqrt{2}$ is ……….
3	The additive inverse of the number $\sqrt{5}-\sqrt{2}$ is …………….
4	The additive inverse of : $\left(\sqrt{7}-\sqrt{3}\right)$ is ……….
5	The additive inverse of $\sqrt{48}-5\sqrt{3}$ is ……….
6	The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\dots\dots\dots}{6}$
7	The multiplicative inverse of $\frac{3}{\sqrt{3}}$ is ……….
8	The multiplicative inverse of $\frac{2}{\sqrt{2}}$ is …………….
9	The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\dots\dots\dots}{\sqrt{3}}$
10	The multiplicative inverse of $\frac{5}{\sqrt{5}}$ is $\frac{\dots\dots\dots\dots\dots}{\sqrt{5}}$
11	📖 If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $a-b$ means the sum of the number a and ………. of the number b
12	📖 If $a \in \mathbb{N}$, $b \in \mathbb{Q}$ and $c \in \mathbb{R}$, then $a+b+c \in$ ……….
13	If : $a-b=2\sqrt{5}$, the value of : $a(a-b)^3+b(b-a)^3=$ ……….
14	$\frac{3\sqrt{2}}{2\sqrt{18}}=$ ……….
15	$\sqrt{3}\times\sqrt{6}=3\times$ ……….
16	$\frac{1}{2}\sqrt{48}=2\times$ ……….
17	If $2\sqrt{27}-2\sqrt{48}=x\sqrt{3}$, then $x=$ ……….
18	📖 $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ………. in the same pattern.
19	If $x^2=\frac{8}{9}$, then x in the simplest form = ……….
20	📖 If $x^2=5$, then $(x+\sqrt{5})^2=$ ………. or ……….
21	If $\sqrt{x}=\sqrt{2}+1$, then $x=$ ……….

EX. (2): Choose the correct answer:

1 The multiplicative inverse of the number $\sqrt{3}$ is ……….

(a) $\frac{3}{\sqrt{3}}$ (b) -3 (c) 3 (d) $\frac{\sqrt{3}}{3}$

2 The multiplicative inverse of the number $\sqrt{7}$ is ……….

(a) $-\sqrt{7}$ (b) $\frac{-1}{\sqrt{7}}$ (c) $\frac{\sqrt{7}}{7}$ (d) $\frac{7}{\sqrt{7}}$

3 $-2\sqrt{3} \times \sqrt{3} =$ ……….

(a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6

4 $\left(2\sqrt[3]{5}\right)^3 =$ ……….

(a) 10   (b) 20   (c) $4\sqrt[3]{5}$ (d) 40

5 The multiplicative inverse of the number $\frac{\sqrt{2}}{10}$ is ……………

(a) $2\sqrt{5}$ (b) $-\frac{\sqrt{9}}{10}$ (c) $\frac{\sqrt{10}}{2}$ (d) $5\sqrt{2}$

6 $2\sqrt{3} + 3\sqrt{3} =$ ……….

(a) $5\sqrt{6}$ (b) $5\sqrt{3}$ (c) $6\sqrt{3}$ (d) $5\sqrt[3]{3}$

7 $5 + 7\sqrt{2} - 4 + \sqrt{2} =$ ……….

(a) 15   (b) $1 + 7\sqrt{2}$ (c) $1 + 8\sqrt{2}$ (d) $1 + 6\sqrt{2}$

8 The multiplicative inverse of the number $\sqrt{5}$ is ……….

(a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

9	The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ……… (a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$
10	The additive inverse of the number $\left(\sqrt{7}-\sqrt{3}\right)$ is …………… (a) $\sqrt{7}+\sqrt{3}$ (b) $-\sqrt{7}+\sqrt{3}$ (c) $\sqrt{3}+\sqrt{7}$ (d) $-\sqrt{7}-\sqrt{3}$
11	The additive inverse of the number $\frac{6}{\sqrt{2}}$ is ……… (a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$
12	The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ is ……… (a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$
13	$\frac{1}{2}\sqrt{20}+10\sqrt{\frac{1}{5}}=$ ……… (a) $3\sqrt{5}$ (b) $4\sqrt{5}$ (c) 5 (d) 12
14	$\sqrt{3}\left(\sqrt{11}+\sqrt{3}\right)=$ ……… (a) $3\sqrt{11}+2$ (b) $\sqrt{33}+3$ (c) $11\sqrt{3}+2$ (d) $2\sqrt{11}+3$
15	$\sqrt{20}-\sqrt{5}=$ …………… (a) $\sqrt{15}$ (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) 5
16	📖 $\sqrt{8}-\sqrt{2}=$ ……… (a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1
17	📖 $\left(\sqrt{8}+\sqrt{2}\right)^2=$ ……… (a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$
18	📖 $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)=$ ……… (a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

EX. (3): Answer the following:

1 Find the result of each of the following in the simplest form :

1. $2\left(\sqrt{2}+\sqrt{5}\right)$
2. $\sqrt{2}\left(5+\sqrt{2}\right)$
3. $\sqrt{7}\left(\sqrt{7}+2\right)$
4. $-\sqrt{3}\left(-5-\sqrt{3}\right)$
5. $\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)$
6. $\sqrt{5}\left(3-\sqrt{5}\right)-2\left(1+\sqrt{5}\right)$

2 Find the simplest form of : $\sqrt{75}-2\sqrt{27}+3\sqrt{\frac{1}{3}}$

3 Find in simplest form the expression : $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$

4 Find in the simplest form : $\sqrt{50}+\sqrt{18}-\sqrt{2}$

5 Simplify to the simplest form : $\sqrt{27}+5\sqrt{18}-\sqrt{300}$

6 Simplify to the simplest form : $2\sqrt{5}+6\sqrt{\frac{1}{3}}-\sqrt{12}-5\sqrt{\frac{1}{5}}$

7 Simplify each of the following to the simplest form :

1. $\sqrt{50}+\sqrt{8}$
2. $\sqrt{20}-\sqrt{45}$

8 Find the value of each of $x+y$, $x \times y$ in each of the following cases :

1. $x=3+\sqrt{5}$, $y=1-\sqrt{5}$
2. $x=\sqrt{3}-\sqrt{2}$, $y=\sqrt{3}+\sqrt{2}$
3. $x=5-3\sqrt{2}$, $y=5-3\sqrt{2}$

Explaining

Lesson (7)
The two Conjugate Numbers

If a and b are two positive rational numbers , then each of the two numbers $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ is conjugate to the other one and we find that

- Their sum $= (\sqrt{a}+\sqrt{b}) + (\sqrt{a}-\sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product $= (\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
  = the square of the first term – the square of the second term.

For example :

$(\sqrt{3}-\sqrt{2})$ its conjugate is $(\sqrt{3}+\sqrt{2})$, then we find that

- Their sum $= 2\sqrt{3}$
- Their product $= 3 - 2 = 1$

Remark

The product of the two conjugate numbers is always a rational number.

Remark

If we have a real number whose denominator is written in the form $(\sqrt{a}+\sqrt{b})$ or $(\sqrt{a}-\sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

Important Remarks

From direct product (multiplying by inspection) ,

- **We know that :** $(x-y)(x+y) = x^2 - y^2$
- **And we know also :**

$(x+y)^2 = x^2 + 2xy + y^2$

Then

- $x^2 + xy + y^2 = (x+y)^2 - xy$
- $x^2 + y^2 = (x+y)^2 - 2xy$

or

$(x-y)^2 = x^2 - 2xy + y^2$

Then

- $x^2 - xy + y^2 = (x-y)^2 + xy$

$x^2 + y^2 = (x-y)^2 + 2xy$

Example 1

If $x = \dfrac{4}{2-\sqrt{2}}$ and $y = \dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}$, write each of x and y such that its denominator is a rational number, then find $x + y$

Solution

$$\because x = \frac{4}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4\left(2+\sqrt{2}\right)}{4-2} = \frac{4\left(2+\sqrt{2}\right)}{2}$$

$$= 2\left(2+\sqrt{2}\right) = 4 + 2\sqrt{2}$$

$$, y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{\left(3-2\sqrt{2}\right)^2}{9-8} = \frac{9-12\sqrt{2}+8}{1} = 17-12\sqrt{2}$$

$$\therefore x + y = 4 + 2\sqrt{2} + 17 - 12\sqrt{2} = 21 - 10\sqrt{2}$$

Example 2

If $x = \dfrac{2}{\sqrt{5}-\sqrt{3}}$ and $y = \sqrt{5}-\sqrt{3}$, prove that x and y are conjugate numbers, then find the value of each of :

$x^2 + 2xy + y^2$

Solution

$$\because x = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2} = \sqrt{5}+\sqrt{3}$$

$$, \because y = \sqrt{5}-\sqrt{3}$$

$\therefore x$ and y are conjugate numbers.

$$x^2 + 2xy + y^2 = \left(\sqrt{5}+\sqrt{3}\right)^2 + 2\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right) + \left(\sqrt{5}-\sqrt{3}\right)^2$$

$$= \left(5 + 2\sqrt{15} + 3\right) + 2\,(5-3) + \left(5 - 2\sqrt{15} + 3\right)$$

$$= 8 + 2\sqrt{15} + 4 + 8 - 2\sqrt{15} = 20$$

Another solution using the previous remarks :

Since $x^2 + 2xy + y^2 = (x+y)^2$

$$\therefore x^2 + 2xy + y^2 = \left[\left(\sqrt{5}+\sqrt{3}\right) + \left(\sqrt{5}-\sqrt{3}\right)\right]^2$$

$$= \left(2\sqrt{5}\right)^2 = 4 \times 5 = 20$$

EX. (1): Complete the following:

Exercises

1	$(\sqrt{2}+\sqrt{3})^{-9}(\sqrt{2}-\sqrt{3})^{-9} =$ ....
2	$(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5}) =$ ....
3	$(\sqrt{3}+2)(\sqrt{3}-2) =$ ....
4	$(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2}) =$ ....
5	$(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) =$ ....
6	$(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2}) =$ ....
7	The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....
8	The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is ....
9	The conjugate of the number $\sqrt{3}-5$ is ....
10	The conjugate number of the number : $\sqrt{5}+\sqrt{3}$ is ....
11	If $x = 3+\sqrt{2}$, then its conjugate is .... and the product of multiplying x by its conjugate is ....
12	The multiplicative inverse for $(\sqrt{3}+\sqrt{2})$ in its simplest form is ....
13	A rectangle of dimensions $(\sqrt{3}-1)$, $(\sqrt{3}+1)$ cm. has an area of .... cm^2
14	If $\frac{x}{5-\sqrt{5}} = 5+\sqrt{5}$, then the value of x in its simplest form is ....

15	If $\frac{1}{x} = \sqrt{5} - 2$, then the value of x in its simplest form is ……….
16	If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(xy , x + y) =$ ……….
17	If $x = \sqrt[3]{3} + 1$ and $y = \sqrt[3]{3} - 1$, then $(x + y)^3 =$ ……….
18	If : $x = 2\sqrt{3} + 3\sqrt{5}$ and $y = 2\sqrt{3} - 3\sqrt{5}$, then $x - y =$ ……….
19	If : $x = \frac{1}{\sqrt{8} - \sqrt{5}}$ and $xy = \frac{1}{3}$, then $y =$ ……….
20	If $x = \sqrt{3} + 1$, $y = \sqrt{3} - 1$, then $(x - y)^2 =$ ……………
21	If $x = 2 + \sqrt{5}$ and y is the conjugate number of x , then $(x - y)^2 =$ ……….

EX. (2): Choose the correct answer:

1	The number $\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)$ is a .... number. (a) natural (b) rational (c) irrational (d) prime
2	The simplest form of the expression : $\left(\sqrt{3}-1\right)^2\left(\sqrt{3}+1\right)^2$ is .... (a) $2\left(\sqrt{3}-1\right)$ (b) $\left(\sqrt{3}+1\right)^2$ (c) 4 (d) 13
3	The multiplicative inverse of $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)$ is .... (a) 4 (b) -4 (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$
4	If : $x=\sqrt{5}+\sqrt{3}$, $y=\sqrt{5}-\sqrt{3}$, then $x-y=$ .... (a) $2\sqrt{3}$ (b) $5\sqrt{3}$ (c) $2\sqrt{5}$ (d) 2
5	If : $x=3+\sqrt{5}$ and $y=3-\sqrt{5}$, then $x-y=$ .... (a) $6\sqrt{5}$ (b) $2\sqrt{5}$ (c) $\sqrt{10}$ (d) 6
6	If : $x^2-y^2=60$ and $x+y=5\sqrt{6}$, then $x-y=$ .... (a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$
7	If $x=3+\sqrt{3}$ and $y=3-\sqrt{3}$, then $x-y=$ .... (a) $6\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{6}$ (d) 6
8	If : $x=\sqrt{7}+\sqrt{3}$, $y=\sqrt{7}-\sqrt{3}$, then $(x-y)^3=$ .... (a) zero (b) 24 (c) $24\sqrt{3}$ (d) 196

EX. (3): Answer the following:

1	**Simplify :** (1) $(4-3\sqrt{2})(4+3\sqrt{2})$ (2) $(\sqrt{3}+2)(\sqrt{3}-1)$
2	If $x=\sqrt{5}+\sqrt{2}$, $y=\frac{3}{\sqrt{5}+\sqrt{2}}$ (1) **Prove that :** x and y are conjugate numbers. (2) **Find the value of :** $\frac{x+y}{xy}$
3	If $a=\sqrt{2}+1$ and $b=\frac{1}{\sqrt{2}+1}$, **find the value of :** $(a-b)^2$
4	If $x=3+\sqrt{5}$ and $y=\frac{4}{3+\sqrt{5}}$, **prove that x and y are conjugate numbers , then find :** 1 Their product. 2 x^2+y^2
5	If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$, find the value of : a^2-b^2 in its simplest form.
6	If $x=\sqrt{5}+\sqrt{2}$ and $y=\sqrt{5}-\sqrt{2}$, **find the value of :** $\frac{x+y}{xy-1}$
7	If $x=\sqrt{5}-\sqrt{3}$ and $y=\frac{2}{\sqrt{5}-\sqrt{3}}$, find the value of : $x^2+2xy+y^2$
8	If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of : x^2y^2
9	If $x=\frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}}$ and $y=\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$, find the value of each of : 1 x^2+y^2 2 xy , then prove that : $x^2+y^2=38xy$

10	If $a = \frac{4}{\sqrt{7}-\sqrt{3}}$ and $b = \frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of : $\frac{a-b}{ab}$
11	If $x = \frac{1}{2+\sqrt{3}}$ and $y = \frac{12}{\sqrt{3}}$, **find the value of :** $x^2 + y$ in its simplest form.
12	If $x = \sqrt{5}-\sqrt{2}$ and $y = \frac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers , then find the value of : $x^2 - 2xy + y^2$

Explaining

Lesson (8)

Operations on The Cube roots

If a and b are two real numbers , then

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$

2 $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)

For example:

- $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\dfrac{32}{4}} = \sqrt[3]{8} = 2$
- $\dfrac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{\dfrac{54}{-2}} = \sqrt[3]{-27} = -3$

Remarks

*** If a and b are two real numbers , then :**

1 $\sqrt[3]{a^3+b^3} \neq a+b$, $\sqrt[3]{a^3-b^3} \neq a-b$

2 $\sqrt[3]{-a} = -\sqrt[3]{a}$

3 $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

For example :

- $3\sqrt[3]{\dfrac{1}{9}} = \sqrt[3]{27 \times \dfrac{1}{9}} = \sqrt[3]{3}$
- $8\sqrt[3]{\dfrac{1}{4}} = 4 \times 2\sqrt[3]{\dfrac{1}{4}} = 4\sqrt[3]{8 \times \dfrac{1}{4}} = 4\sqrt[3]{2}$

4 $\sqrt[3]{\dfrac{a}{b}} = \sqrt[3]{\dfrac{a}{b} \times \dfrac{b^2}{b^2}} = \sqrt[3]{\dfrac{a\,b^2}{b^3}} = \dfrac{1}{b}\sqrt[3]{a\,b^2}$

For example :

- $\sqrt[3]{\dfrac{1}{3}} = \sqrt[3]{\dfrac{1}{3} \times \dfrac{9}{9}} = \sqrt[3]{\dfrac{9}{27}} = \dfrac{1}{3}\sqrt[3]{9}$

Important Remark

$\sqrt[3]{16} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$

$\sqrt[3]{54} = \sqrt[3]{27} \times \sqrt[3]{2} = 3\sqrt[3]{2}$

$\sqrt[3]{128} = \sqrt[3]{64} \times \sqrt[3]{2} = 4\sqrt[3]{2}$

$\sqrt[3]{250} = \sqrt[3]{125} \times \sqrt[3]{2} = 5\sqrt[3]{2}$

$\sqrt[3]{24} = \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

$\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$

$\sqrt[3]{40} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$

$\sqrt[3]{135} = \sqrt[3]{27} \times \sqrt[3]{5} = 3\sqrt[3]{5}$

Example 1 Find the result of each of the following in its simplest form :

1) $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}}$

2) $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}}$

Solution

1) $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{2}{3} \times \frac{4}{9}} = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

2) $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}} = \sqrt[3]{\frac{5}{4} \div \frac{2}{25}} = \sqrt[3]{\frac{5}{4} \times \frac{25}{2}} = \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$

! Remarks

If a and b are two real numbers , then :

1. $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$
2. $\sqrt[3]{-a} = -\sqrt[3]{a}$
3. $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

   For example: • $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$

   • $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

4. $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{a\,b^2}{b^3}} = \frac{1}{b}\sqrt[3]{a\,b^2}$ (Where $b \neq 0$)

   For example: $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Example 2 Put each of the following in its simplest form :

1) $\sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81}$

2) $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$

3) $\sqrt[3]{81} + \sqrt{12} - 2\sqrt[3]{3} - 2\sqrt{3}$

Solution

(1) $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}=\sqrt[3]{8\times 3}+\sqrt[3]{3}-\sqrt[3]{27\times 3}$

$=\sqrt[3]{8}\times\sqrt[3]{3}+\sqrt[3]{3}-\sqrt[3]{27}\times\sqrt[3]{3}$

$=2\sqrt[3]{3}+\sqrt[3]{3}-3\sqrt[3]{3}=\text{zero}$

(2) $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{27\times 2}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$

$=\sqrt[3]{27}\times\sqrt[3]{2}+6\times\sqrt[3]{8}\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}$

$=3\times\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

Another solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{16}{16}}=\sqrt[3]{\frac{16}{64}}=\frac{\sqrt[3]{16}}{\sqrt[3]{64}}=\frac{1}{4}\sqrt[3]{16}$

$=\frac{1}{4}\sqrt[3]{8\times 2}=\frac{1}{4}\times 2\sqrt[3]{2}=\frac{1}{2}\sqrt[3]{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+6\times 2\sqrt[3]{2}-6\times\frac{1}{2}\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

One more solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{2}{2}}=\sqrt[3]{\frac{2}{8}}=\frac{\sqrt[3]{2}}{\sqrt[3]{8}}=\frac{\sqrt[3]{2}}{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-6\times\frac{\sqrt[3]{2}}{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

(3) $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{27\times 3}+\sqrt{4\times 3}-2\sqrt[3]{3}-2\sqrt{3}$

$=\sqrt[3]{27}\times\sqrt[3]{3}+\sqrt{4}\times\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}$

$=3\sqrt[3]{3}+2\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{3}$

EX. (1): Complete the following:

Exercises

1	$\sqrt[3]{2}+\sqrt[3]{2}=\sqrt[3]{\cdots\cdots\cdots}$
2	If : $x=\sqrt[3]{3}+7$, $y=\sqrt[3]{3}-7$, then $(x+y)^3=\cdots\cdots\cdots$
3	If : $x=\sqrt[3]{3}+1$, $y=\sqrt[3]{3}-1$, then $(x+y)^3=\cdots\cdots\cdots$
4	$\sqrt[3]{\frac{2}{3}}\times\sqrt[3]{-12}=\cdots\cdots\cdots$
5	$\sqrt[3]{54}-\sqrt[3]{-16}=\sqrt[3]{\cdots\cdots\cdots}$
6	The multiplicative neutral in $\mathbb{R}$ is ……… and the additive neutral in $\mathbb{R}$ is ………
7	The additive inverse of the number $1-\sqrt{2}$ is ………
8	The multiplicative inverse of $\frac{2}{\sqrt{2}}$ is ……………
9	If $a\in\mathbb{R}$ and $b\in\mathbb{R}$, then $a-b$ means the sum of the number a and ……… of the number b
10	If $a\in\mathbb{N}$, $b\in\mathbb{Q}$ and $c\in\mathbb{R}$, then $a+b+c\in$………
11	If : $a-b=2\sqrt{5}$, the value of : $a(a-b)^3+b(b-a)^3=\cdots\cdots\cdots$
12	$\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt{\cdots\cdots\cdots}$
13	If $x=2$, $y=\sqrt[3]{-16}$, then $\left(\frac{x}{y}\right)^3=\cdots\cdots\cdots$
14	$\frac{1}{2}\sqrt[3]{56}-\sqrt[3]{\frac{7}{27}}=\cdots\cdots\cdots$

EX. (2): Choose the correct answer:

1	$\sqrt[3]{(-8)^2} = \cdots\cdots\cdots$ (a) 2 (b) -2 (c) 4 (d) -4
2	If : $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(x y , x + y) = \cdots\cdots\cdots$ (a) $(-1 , 2\sqrt{3})$ (b) $(1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) $(-1 , 4)$
3	$\sqrt[3]{\frac{2}{9}} = \cdots\cdots\cdots$ (a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$
4	$\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \cdots\cdots\cdots$ (a) 8 (b) -2 (c) 2 (d) $2\sqrt[3]{2}$
5	$\sqrt[3]{24} + \sqrt[3]{-81} + \sqrt[3]{3} = \cdots\cdots\cdots$ (a) $\sqrt[3]{3}$ (b) 0 (c) $6\sqrt[3]{3}$ (d) $-\sqrt[3]{3}$
6	🕮 $\sqrt[3]{54} + \sqrt[3]{-2} = \cdots\cdots\cdots$ (a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$
7	🕮 $\sqrt[3]{-64} + \sqrt{16} = \cdots\cdots\cdots$ (a) zero (b) 8 (c) -8 (d) ± 8
8	$\sqrt[3]{2} + \sqrt[3]{2} = \cdots\cdots\cdots$ (a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$
9	The number $(1 - \sqrt{3})(1 + \sqrt{3})$ is a …………… number. (a) natural (b) rational (c) irrational (d) prime

EX. (3): Answer the following:

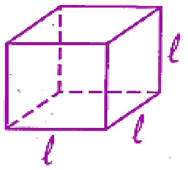
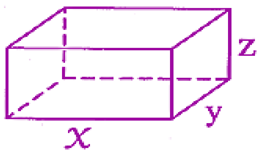
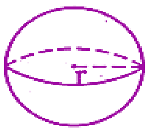
1	Find in simplest form : $\sqrt[3]{54} - \sqrt[3]{16} + \sqrt[3]{2}$
2	Simplify : $\sqrt[3]{54} + \sqrt[3]{2} + \sqrt[3]{-128}$
3	Simplify to the simplest form : $3\sqrt{12} + \sqrt[3]{54} - 2\sqrt{27} - \sqrt[3]{16}$
4	Prove that : $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$
5	Find the value of : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$
6	Simplify to the simplest form : $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$
7	If $a = \sqrt[3]{5} + 1$, $b = \sqrt[3]{5} - 1$, find the value of each of the following : 1 $(a - b)^5$ 2 $(a + b)^3$
8	Prove that : 1 $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = \text{zero}$ 2 $\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6) = 1$
9	If $x = 3 + \sqrt[3]{6}$, $y = 3 - \sqrt[3]{6}$, find the value of : $\left(\frac{x-y}{x+y}\right)^3$
10	Find the result of each of the following in its simplest form : 1 $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$ 2 $\sqrt[3]{125} - \sqrt[3]{24}$
11	Find the result of each of the following in its simplest form : 1 $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$ 2 $\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$ 3 $\sqrt[3]{\frac{3}{4}} \div \sqrt[3]{\frac{2}{9}}$

Explaining

Lesson (9)

Applications on the Real Numbers

In the following , we will summarize the previous rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere		–	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Exercises

EX. (1): Complete the following:

1	If the side length of a square is ℓ cm. and its area is 30 cm^2, then the area of the square whose side length equals 2ℓ cm. is ....
2	Area of the square of side length is (2 l) cm. = ... cm^2
3	The lateral area of a cube whose edge length is ℓ cm. = ... cm^2
4	📖 The edge length of a cube is 4 cm. , then its total area = ... cm^2

5	If the edge length of a cube is 5 cm. , then its volume = ... cm^3.
6	The cube whose edge length is 2 ℓ cm. , then its volume = ... cm^3.
7	If the volume of a cube is 216 cm^3 , then the length of its edge is ....
8	A cube whose volume is 1000 cm^3 , then its side length = ...
9	If the volume of a cube is 27 cm^3 , then its lateral area is .... cm^2.
10	The cube whose volume is ℓ^3 cm^3 , its total area = ... cm^2.
11	The sum of lengths of all edges of a cube is 36 cm. , then its total area equals .... cm^2.
12	The volume of a cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. , $\sqrt{6}$ cm. is ....
13	The volume of right circular cylinder 90 π cm^3. and its height is 10 cm. then the radius length of its base equal .... cm.
14	The volume of the sphere = ...
15	The radius length of the sphere whose volume is $\frac{4}{3}$ π cm^3. = ...
16	The volume of the sphere whose diameter length is 6 cm. = ... π cm^3.
17	If the volume of a sphere is $\frac{9}{2}$ π cm^3 , then its radius length = ... cm.
18	If the volume of the sphere is $\frac{9}{16}$ π cm^3 , then its radius length = ... cm.

EX. (2): Choose the correct answer:

1	The volume of a sphere which its diameter 6 cm. = ……… cm^3 (a) 4π (b) 9π (c) 36π (d) 27π
2	If the radius length of a sphere is 3 cm. , then its volume is …………… (a) $4\pi\ cm^3$ (b) $9\pi\ cm^3$ (c) $27\pi\ cm^3$ (d) $36\pi\ cm^3$
3	The volume of the sphere equals $32\sqrt{3}\,\pi\ cm^3$, then its radius length = ……… (a) $\sqrt{3}$ cm. (b) 3 cm. (c) $2\sqrt{3}$ cm. (d) 9 cm.
4	If the volume of a sphere is $\frac{9}{16}\pi\ cm^3$, then its radius length is …………… (a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$
5	If the volume of a sphere is $\frac{4}{3}\pi\ cm^3$, then its radius length = …………… cm. (a) 2 (b) 1 (c) 8 (d) 27
6	The volume of a sphere is $\frac{9}{2}\pi\ cm^3$, then its radius length = …………… cm. (a) 1 (b) 1.5 (c) 2 (d) 3
7	The lateral surface area of right circular cylinder = ……… (a) $\pi r h$ (b) $4\pi r^2$ (c) $\pi r^2 h$ (d) $2\pi r h$
8	The radius length of a right circular cylinder whose volume is $40\pi\ cm^3$ and its height 10 cm. = ……… cm. (a) 5 (b) 3 (c) 2 (d) 1
9	The volume of a right circular cylinder is $90\pi\ cm^3$ and its height is 10 cm. , then the radius length of its base = …………… cm. (a) 3 (b) 4.5 (c) 5 (d) 9

10	Volume of a cube whose edge length 2ℓ cm. is .... cm^3 (a) $2\ell^3$ (b) 8ℓ (c) $8\ell^3$ (d) ℓ^3
11	The volume of a cube whose edge length is 2 cm. is .... cm^3 (a) 8 (b) 4 (c) 16 (d) 6
12	If the area of one face of a cube is 25 cm^2 , then it's volume = ... (a) 125 cm^3 (b) 50 cm^2 (c) 5 cm^3 (d) 625 cm^3
13	The edge length of a cube whose volume is $2\sqrt{2}$ cm^3 = ... cm. (a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5
14	The cube whose volume is $3\sqrt{3}$ cm^3 , its side length is .... cm. (a) 3 (b) $\sqrt{3}$ (c) 9 (d) 27
15	The edge length of a cube whose volume is 3 m^3 is .... cm. (a) $\sqrt{3}$ (b) 3 (c) 1 (d) $\sqrt[3]{3}$
16	The cube whose volume is 8 cm^3 , then its total area = ... cm^2 (a) 16 (b) 24 (c) 96 (d) 4
17	If the volume of a cube is 27 cm^3 , then the total area is .... cm^2 (a) 54 (b) 9 (c) 27 (d) 36
18	If the volume of a cube is $40\sqrt{5}$ cm^3 , then its edge length is .... cm. (a) $\sqrt{5}$ (b) $8\sqrt{5}$ (c) $2\sqrt{5}$ (d) $5\sqrt{2}$
19	The volume of a cube is 64 cm^3 , then its total area = ... (a) 64 cm^2 (b) 96 cm^2 (c) 36 cm^2 (d) 24 cm^2
20	A cube of volume 216 cm^3 has a total area = ... cm^2 (a) 6 (b) 36 (c) 144 (d) 216

EX. (3): Answer the following:

1 A cube whose lateral area is 36 cm^2. Find :

1 Its total area. 2 Its volume.

2 The perimeter of one face of a cube is 12 cm. Find :

1 Its volume. 2 Its lateral area.

3 The sum of lengths of all edges of a cube is 60 cm. Find :

1 Its volume. 2 Its total area.

4 Find the total area of a cuboid whose volume is 720 cm^3 and its height is 5 cm. with a squared-shape base.

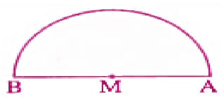
5 Which is more in size :

A cube whose total area is 294 cm^2 or a cuboid with dimensions $7\sqrt{2}$ cm. , $5\sqrt{2}$ cm. and 5 cm. ?

6 A circle whose area is 64π cm^2. Find the length of its radius , then find its circumference approximating it to the nearest integer. ($\pi = 3.14$)

7 In the opposite figure :

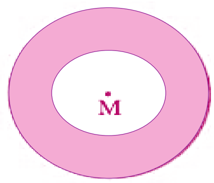
$\overline{AB}$ is a diameter of the semicircle. If the area of this region is 12.32 cm^2, find the perimeter of the figure.



8 In the opposite figure :

These are two concentric circles at M and their radii lengths are 3 cm. and 5 cm.

Find the area of the shaded part in terms of π



9 Find the lateral area for a right circular cylinder of volume 924 cm^3. , and of a height 6 cm. $\pi = \frac{22}{7}$

10 Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm. ($\pi = 3.14$)

11	The volume of a sphere is $\frac{99000}{7}$ cm^3. Calculate its radius length. $\left(\pi = \frac{22}{7}\right)$
12	If the volume of a sphere is $\frac{32}{3}\pi$ cm^3, find the length of its diameter.
13	Find the radius length of a sphere whose volume is 36π cm^3.
14	If the volume of a right circular cylinder is 90π cm^3 and its height is 10 cm. find the radius length.
15	A right circular cylinder, the radius length of its base is 3.5 cm. and its height 10 cm. Find the volume of the cylinder. $(\pi = 3.14)$
16	A metal cuboid with dimensions 77 cm., 24 cm., 21 cm. It was melted to make a sphere, find the radius length of that sphere. $\left(\pi = \frac{22}{7}\right)$
17	Find the volume and the total area of a right circular cylinder in which the radius length of the base = 14 cm. and the height is 20 cm.
18	The volume of the sphere is 36π cm^3. Find its radius length.
19	If $\frac{3}{4}$ the volume of a sphere is 8π cm^3, find its radius length.
20	Find the surface area of a sphere if its radius length = 7 cm. $\left(\pi = \frac{22}{7}\right)$
21	The right circular cylinder its radius length is 7 cm. and its height is 15 cm. **Find the lateral area.** $\left(\pi = \frac{22}{7}\right)$

Explaining

Lesson (10)

Solving equations and inequalities of the first degree in one variable in R

For example: **Find the S.S of the following in R:**

1) $2X = 10$	2) $X - 3 = 5$	3) $2X - 3 = 11$	4) $3X + 1 = 13$
$X = 10 \div 2$ $X = 5$ $S.S = \{5\}$	$X = 5 + 3$ $X = 8$ $S.S = \{8\}$	$X = \frac{11+3}{2}$ $X = 7$ $S.S = \{7\}$	$X = \frac{13-1}{3}$ $X = 4$ $S.S = \{4\}$
5) $X + 5 \geq 8$	6) $2X > 10$	7) $4X + 1 \leq 21$	8) $5X - 4 < 26$
$X \geq 8 - 5$ $X \geq 3$ $[3, \infty[$	$X > 10 \div 2$ $X > 5$ $]5, \infty[$	$X \leq \frac{21-1}{4}$ $X \leq 5$ $]-\infty, 5]$	$X < \frac{26+4}{5}$ $X < 6$ $]-\infty, 6[$
9) $3 < X + 1 \leq 5$	10) $7 < 3X + 1 \leq 22$	11) $5 \leq 3 - 2X \leq 7$	12) $-2 \leq 3X + 7 < 22$
$3 - 1 < X \leq 5 - 1$ $2 < X \leq 4$ $]2, 4]$	$\frac{7-1}{3} < X \leq \frac{22-1}{3}$ $2 < X \leq 7$ $]2, 7]$	$\frac{5-3}{-2} \geq X \geq \frac{7-3}{-2}$ $-1 \geq X \geq -2$ $[-2, -1]$	$\frac{-2-7}{3} \leq X < \frac{22-7}{3}$ $-3 \leq X < 5$ $[-3, 5[$
13) $3 + X < 2X < 7 + X$	14) $3 + X \leq 2X + 1 < 7 + X$	15) $3 - X < 2X \leq 12 - X$	16) $14 - X \leq 5X + 2 \leq 26 - X$
$3 < 2X - X < 7$ $3 < X < 7$ $]3, 7[$	$3 \leq 2X - X + 1 < 7$ $3 \leq X + 1 < 7$ $3 - 1 \leq X < 7 - 1$ $2 \leq X < 6$ $[2, 6[$	$3 < 2X + X \leq 12$ $3 < 3X \leq 12$ $3 \div 3 < 3X \leq 12 \div 3$ $1 < X \leq 4$ $]1, 4]$	$14 \leq 5X + X + 2 \leq 26$ $14 \leq 6X + 2 \leq 26$ $\frac{14-2}{6} \leq X \leq \frac{26-2}{6}$ $2 \leq X \leq 4$ $[2, 4]$

EX. (1): Complete the following:

Exercises

1	The set $X = \{x : x \geq 2 , x \in \mathbb{R}\}$ in the form of an interval is ……………
2	The solution set of the equation : $x + \sqrt{2} = \sqrt{8}$ in $\mathbb{R}$ is ………
3	If : $x + 2\sqrt{3} = 3$, then $x =$ ………
4	The S.S. of the equation $\sqrt{2}\, x - 1 = 1$ is ……… where $x \in \mathbb{R}$
5	The S.S. of the inequality : $-x + 1 \leq 0$ in $\mathbb{R}$ is ………
6	If $x - 3 \geq 0$, then x ………
7	If $5\,x < 15$, then x ………
8	If $1 - x > 4$, then x ………
9	If $-2\,x \leq 3$, then x ………
10	If $\sqrt{2}\,x \leq 4$, then x ………
11	The S.S. of the inequality : $4 < 2\,x < 8$ in $\mathbb{R}$ is ………
12	The S.S. of the inequality : $-5 \leq -x < 2$ in $\mathbb{R}$ is ………
13	The S.S. of the inequality : $2 - x < 0$ in $\mathbb{R}$ is ………
14	If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2\,x \in]-6$, ……… [
15	If $-x < 2$, then $x \in$ ………
16	The S.S. of the inequality : $-x + 1 \leq 0$ in $\mathbb{R}$ is ………

EX. (2): Choose the correct answer:

1 The S.S. of the inequality : $0 < x + 5 \le 6$ in $\mathbb{R}$ is ……………

(a) $]5 , 11]$ (b) $]-1 , 5]$ (c) $[-5 , 1[$ (d) $]-5 , 1]$

2 The S.S. of the inequality : $-x > 2$ in $\mathbb{R}$ is ……………

(a) $\{2\}$ (b) $]-\infty , 2[$ (c) $]2 , \infty[$ (d) $]-\infty , -2[$

3 If $-1 < -x \le 5$, then the S.S. in $\mathbb{R}$ is ……………

(a) $[-5 , 1[$ (b) $[5 , -1[$ (c) $]-5 , 1]$ (d) $]-5 , 1[$

4 $[-3 , 7] - \{-3 , 7\} =$ ……………

(a) $[-3 , 7[$ (b) $]-3 , 7]$ (c) $]-3 , 7[$ (d) $[-3 , 7]$

5 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ……………

(a) $\{3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

6 The S.S. of equation : $\sqrt{2}\, x = 2$ in $\mathbb{R}$ is ……….

(a) $\{\sqrt{2}\}$ (b) $\sqrt{2}$ (c) $\{2\}$ (d) $\{2\sqrt{2}\}$

7 $\{x : x \in \mathbb{R} , x < 1\} =$ ……………

(a) $0 , -1 , -2 , \cdots$ (b) $]-\infty , 1]$ (c) $]-\infty , 1[$ (d) $]-\infty , 0]$

8 If : $x \in \mathbb{R}$, $1 - 7x > |-8|$, then $x <$ ……………

(a) 1 (b) -1 (c) $\frac{9}{7}$ (d) 0

9 If : $2 < x < 5$, then $3x - 1 \in$ ……………

(a) $]3 , 12[$ (b) $]6 , 14[$ (c) $]5 , 15[$ (d) $]5 , 14[$

10	The solution set for the equation : $x^2 = 2$ in $\mathbb{R}$ is .... (a) $\{\sqrt{2}\}$ (b) $\{-\sqrt{2}\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\{2\}$
11	If : $-2x > -6$, then $x \in$ .... (a) $]-\infty, 3[$ (b) $]3, \infty[$ (c) $]-2, -6[$ (d) $]1, 3[$
12	The solution set in $\mathbb{R}$ of the inequality : $-x < 3$ is .... (a) $]-\infty, 3[$ (b) $]3, \infty[$ (c) $]-\infty, -3[$ (d) $]-3, \infty[$
13	The S.S. of the inequality : $-x > 2$ in $\mathbb{R}$ is .... (a) $]-\infty, 2[$ (b) $]2, \infty[$ (c) $]-\infty, -2[$ (d) $\{2\}$
14	The S.S. of the equation : $x^2 + 3 = 0$ in $\mathbb{R}$ is .... (a) $\varnothing$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $\pm\sqrt{3}$
15	If the S.S of the inequality : $-1 < x + 3 < 3$ in $\mathbb{R}$ is .... (a) $[-4, 0]$ (b) $[2, 6]$ (c) $]2, 6[$ (d) $]-4, 0[$
16	The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is .... (a) $\{-3\}$ (b) $]3, \infty[$ (c) $]-\infty, 3[$ (d) $]-\infty, -3[$
17	The S.S. in $\mathbb{R}$ of the equation : $x^3 + 11 = 12$ in $\mathbb{R}$ is .... (a) $\{11\}$ (b) $\{12\}$ (c) $\{1\}$ (d) $\{3\}$
18	The S.S. of the equation : $x^3 + 27 = 0$ in $\mathbb{R}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3\sqrt{3}\}$ (d) $\{\pm 3\sqrt{3}\}$
19	The S.S. of the equation : $x^3 + 8 = 0$ in $\mathbb{R}$ is .... (a) $\{2\}$ (b) $\{2\sqrt{2}\}$ (c) $\{-2\}$ (d) $\{2, -2\}$

EX. (3): Answer the following:

1	Find in $\mathbb{R}$ the S.S. of : (1) $5x + 6 = 1$ (2) $3 < x + 2 \le 6$
2	Find in $\mathbb{R}$ the S.S. of : $4 < 3x + 1 \le 10$ and represent it on the number line.
3	Find the S.S. of the inequality : $5x - 3 < 2x + 9$ in $\mathbb{R}$, then represent it on the number line.
4	Find the S.S. of the inequality : $\lvert -3 \rvert < 2x - 1 < 5$ in $\mathbb{R}$ by using number line
5	Find in $\mathbb{R}$ the S.S. of the inequality : $4x \le 5x + 2 < 4x + 3$
6	Find the S.S. in $\mathbb{R}$: $1 - x \ge -2x - 3$
7	Find the S.S. of the inequality : $x - 1 < 3x - 1 \le x + 1$ in $\mathbb{R}$, then represent it on the number line.
8	Find the S.S. of the inequality : $2 + 2x \le 3x + 3 < 5 + 2x$ in $\mathbb{R}$, then represent the interval of the solution on the number line.
9	Find the S.S. of the inequality : $1 < 5 - x \le 3$ $\mathbb{R}$, then represent the interval of the solution on the number line.
10	Find the solution set for the inequality : $\sqrt[3]{-8} \le x + 1 \le \sqrt{9}$ in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line.
11	Find in $\mathbb{R}$ the S.S. of the inequality : $x + 3 \ge 2x \ge x - 2$ and represent it on the number line.
12	Find the S.S. of the inequality : $\frac{3x-4}{6} < x + 1 < \frac{x+3}{2}$ in $\mathbb{R}$, then represent the interval of the solution on the number line.

Explaining

Lesson (1)
The Relation between two variables

The linear relation

- It is a relation of the first degree between two variables x and y , it is in the form $\boxed{a x + b y = c}$, where a , b and c are real numbers , a and b are not both equal to zero
- There is an infinite number of ordered pairs which satisfy this relation.
- If we represent it graphically , the graph will be a straight line therefore it is called a linear relation , this will be shown later when we study the graphic representation of the linear relation.

The graphic representation of the linear relation

Example 5 **Represent the relation :** $2x - y = 3$ graphically

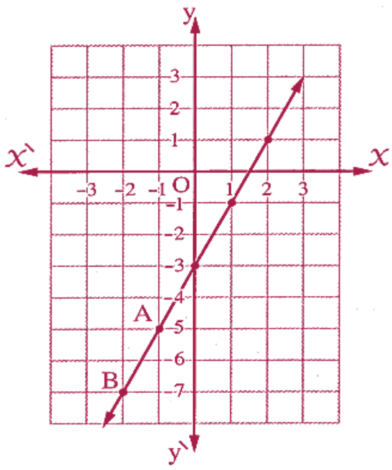
Solution

To represent this relation graphically, we should determine three ordered pairs satisfying the relation : $2x - y = 3$, as follows :

- Set $x = 0$ $\therefore 2 \times 0 - y = 3$ $\therefore -y = 3$ $\therefore y = -3$
- Set $x = 1$ $\therefore 2 \times 1 - y = 3$ $\therefore -y = 1$ $\therefore y = -1$
- Set $x = 2$ $\therefore 2 \times 2 - y = 3$ $\therefore -y = -1$ $\therefore y = 1$

It is preferable to put the values of x and y in a table as the following :

x	0	1	2
y	− 3	− 1	1



Then we determine the points which represent these ordered pairs : (0 , − 3) , (1 , − 1) and (2 , 1) on orthogonal coordinates system , then we draw the straight line passing through these points , it will be the graphic representation of the relation : $2x - y = 3$

Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair (− 1 , − 5) which satisfies the relation when we put $x = -1$ we find that $2 \times (-1) - y = 3$ **i.e.** $y = -5$ and also the point B (− 2 , − 7)

Special cases

We studied before the relation : $a\,x + b\,y = c$, where a , b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

1 If $a = 0$, $b \neq 0$

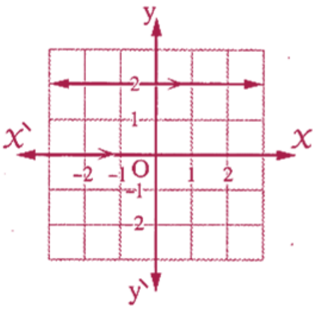
Then the relation becomes in the form :

$b\,y = c$

and it is represented graphically by a straight line parallel to x-axis and intersects y-axis at the point $\left(0 , \frac{c}{b}\right)$

For example:

The relation : $2\,y = 4$ i.e. $y = 2$ is represented by a straight line parallel to x-axis and intersects y-axis at the point $(0 , 2)$



Notice that :

The relation : $y = 0$ is represented by x-axis

2 If $b = 0$, $a \neq 0$

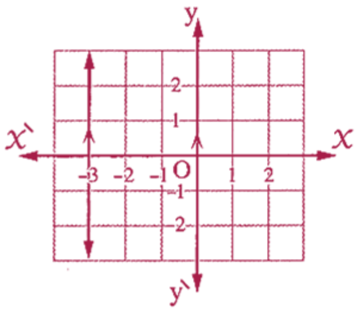
Then the relation becomes in the form :

$a\,x = c$

and it is represented graphically by a straight line parallel to y-axis and intersects x-axis at the point $\left(\frac{c}{a} , 0\right)$

For example:

The relation : $x = -3$ is represented by a straight line parallel to y-axis and intersects x-axis at the point $(-3 , 0)$



Notice that :

The relation : $x = 0$ is represented by y-axis

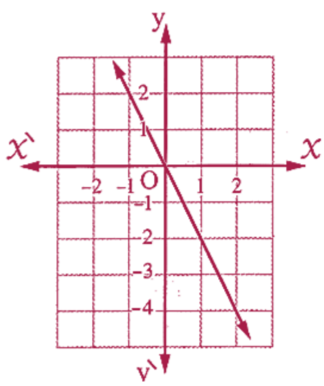
3 If $c = 0$

Then the relation becomes : $a\,x + b\,y = 0$ and it is represented by a straight line passing through the origin point.

For example:

The relation : $2\,x + y = 0$ is represented graphically by a straight line passing through the origin point as shown in the opposite graph :

x	1	-1	2
y	-2	2	-4



Explaining

Lesson (2)

The Slope of Straight line

If A and B are two points in the coordinates plane where A (x_1 , y_1) and B (x_2 , y_2) , then :

The slope of the straight line $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$

Definition

The slope of the straight line $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \dfrac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. • $S = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$ • S is undefined if $x_1 = x_2$

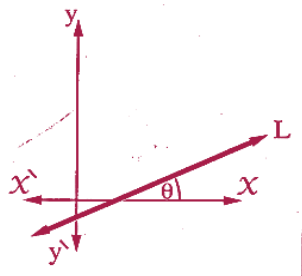
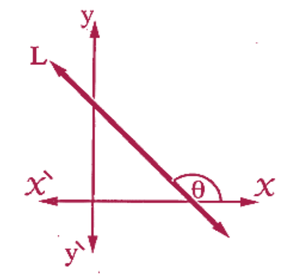
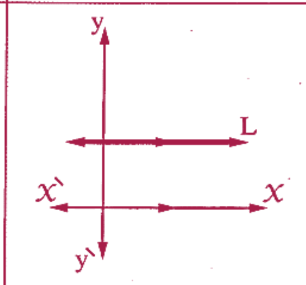
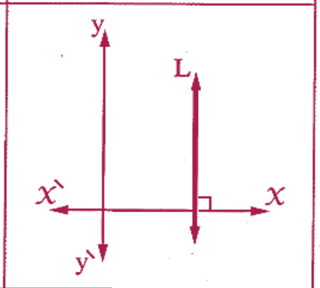
Notice that

The straight line passes through the two points (2 , 0) and (7 , 5) , then :

the slope of the straight line $L = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-0}{7-2} = \dfrac{5}{5} = 1$

Remark

The angle which the straight line L makes with the positive direction of the x-axis takes one of the following cases :

1 Acute angle	2 Obtuse angle	3 Zero angle	4 Right angle
			
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

Example **Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.**

Solution

$\because S = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \dfrac{2-3}{4-2} = -\dfrac{1}{2}$, the slope of $\overleftrightarrow{BC} = \dfrac{0-2}{8-4} = \dfrac{-2}{4} = -\dfrac{1}{2}$

, $\because$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is common.

$\therefore$ The points A , B and C are collinear.

Explaining

Lesson (3)

Real life Applications on the Slope

- We studied before that if there is a linear relation between two variables x and y , then :

The slope of the straight line which represents this relation $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

i.e. The slope of the straight line (S) expresses the rate of change of y with respect to x

- In our life , there are many applications which we need to know the rate of change in dealing with them.

For example:

1 If the opposite graph represents the motion of a car , then :

The uniform velocity of the car (v)

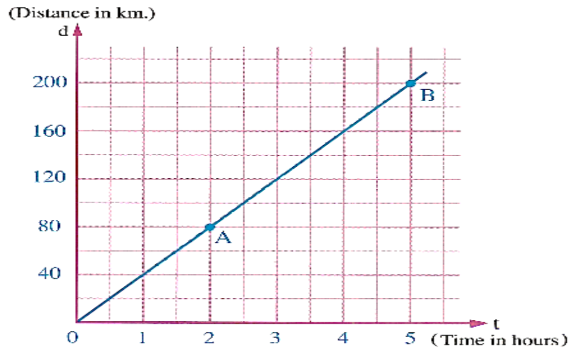
= the rate of change of the distance (d) with respect to the time (t)

i.e. The uniform velocity of the car (v)

= the slope of the straight line (S)

and by selecting two points on the straight line as A (2 , 80) and B (5 , 200)

$\therefore\ v = \dfrac{d_2 - d_1}{t_2 - t_1} = \dfrac{200 - 80}{5 - 2} = \dfrac{120}{3} = 40$ km./hr.



2 If the opposite graph represents the change in the capital of a company (y) within the time (t) , then :

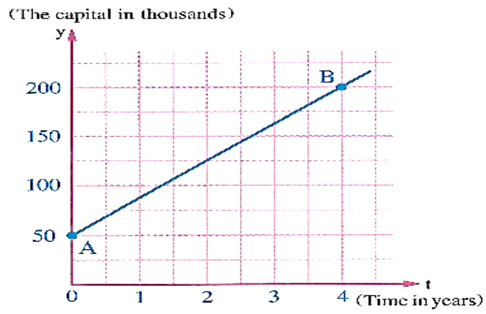
The rate of change in the capital of the company

= the slope of the straight line $\overleftrightarrow{AB}$

$\therefore$ The rate of change of the capital of the company

$= \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{200 - 50}{4 - 0}$

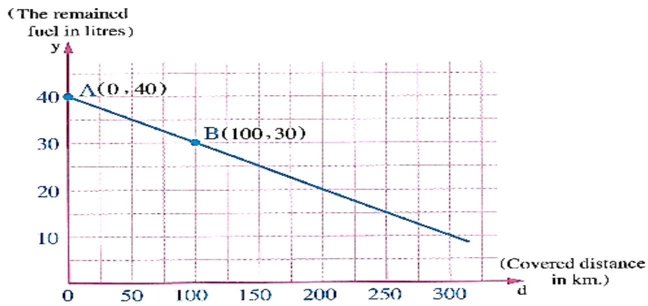
$= \dfrac{150}{4} = 37.5$ thousand pounds / year.



i.e. The capital of the company increases in the rate $= 37.5 \times 1000 = 37500$ pounds/year.

3 A person filled the tank of his car whose capacity is 40 litres with fuel. After he covered a distance 100 km. , he found that the remained fuel in the tank = 30 litres.

The opposite figure shows the relation between the covered distance in km. (d) and the amount of the remained fuel in the tank in litres (y) , then :



The rate of consumption of fuel = the slope of $\overleftrightarrow{AB}$

i.e. The rate of consumption of fuel $= \dfrac{y_2 - y_1}{d_2 - d_1} = \dfrac{30 - 40}{100 - 0} = \dfrac{-10}{100} = -\dfrac{1}{10}$ litre/km.

(The negative sign denotes the amount of fuel decreases in the rate of one litre for each 10 km.)

EX. (1): Choose the correct answer:

Exercises

1	Which of the following represent linear relation ? A) $Xy = 2$ B) $X^2 = \frac{1}{y}$ C) $\frac{X}{y} = 1$ D) $y = X^2 + 4$
2	Which of the following satisfies the relation : $2X + y = 5$? A) $(-3, 3)$ B) $(1, 3)$ C) $(3, 1)$ D) $(2, 2)$
3	$(3, 2)$ satisfies the relation .... A) $Y + X = 5$ B) $Y - X = 5$ C) $3Y - X = 2$ D) $2X + Y = 1$
4	$(3, 2)$ does not satisfy the relation .... A) $Y + X = 5$ B) $X - Y = 1$ C) $Y + X = 7$ D) $3Y - X = 3$
5	Value of b where $(-3, 2)$ satisfies the relation : $3X + by = 1$ is A) 3 B) 5 C) 4 D) 0
6	If : $(2, -5)$ satisfies the relation : $3X - Y + c = 0$, then c = ... A) 1 B) -1 C) 11 D) -11
7	If : $(-1, 5)$ satisfies the relation : $3X + ky = 7$, then k = ... A) 2 B) -2 C) 1 D) 10
8	If : $(a, 1)$ satisfies the relation : $2X + 3y = 7$, then a = ... A) 2 B) -2 C) 4 D) 3
9	If : $(2, b)$ satisfies the relation : $3X + y = 9$, then b = ... A) 6 B) 3 C) 2 D) 0
10	If : $(a, 4)$ satisfies the relation : $X - y = -1$, then a = ... A) $\sqrt{3}$ B) 3 C) 27 D) 1.5
11	If : $(a, 2a)$ satisfies the relation : $y = X - 1$, then a = ... A) 1 B) 10 C) -1 D) 3
12	If : $(k, 2k)$ satisfies the relation : $3X + 2y = 14$, then k = ... A) 2 B) -2 C) 7 D) 0

13 If : (2 k , 3 k) satisfies the relation : X + y = 15 , then k = ……………

A) 5 B) 3 C) - 5 D) - 3

14 If : (2 m , m) satisfies the relation : 2 X + 3 y = 35 , then m = ……………

A) 7 B) 5 C) 14 D) 10

15 The opposite table shows the relation between x and y , which is ……….

x	1	2	3	4	5
y	1	3	5	7	9

(a) $y = x + 4$ (b) $y = x + 1$

(c) $y = 2x - 1$ (d) $y = 3x - 2$

16 The slope of the straight line parallel to the X – axis is ……………

A) Positive B) Negative C) Zero D) Undefined

17 The slope of the straight line parallel to the Y – axis is ……………

A) Positive B) Negative C) Zero D) Undefined

18 The slope of horizontal line is ……………

A) 1 B) Zero C) - 1 D) Undefined

19 If A (1 , 2) , B (0 , 4) , then the slope of $\overleftrightarrow{AB}$ = ……………………

A) - 2 B) 2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

20 Slope of straight line passes through (- 2 , 3) and (2 , 3) is …………

A) 2 B) 1 C) Zero D) Undefined

21 Slope of straight line passes through (- 3 , 1) and (2 , 5) is …………

A) $\frac{4}{5}$ B) $-\frac{6}{1}$ C) $\frac{5}{4}$ D) $-\frac{1}{6}$

22 Slope of straight line passes through (3 , 2) and (– 5 , 3) is …………

A) $\frac{1}{8}$ B) $-\frac{1}{8}$ C) 8 D) – 8

23 If A (3 , 5) , B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ……………………

A) – 3 B) $-\frac{1}{3}$ C) 3 D) $\frac{1}{3}$

24 Slope of straight line passes through (3 , 8) and (0 , 2) is …………

A) 1 B) 2 C) 1 D) $\frac{1}{2}$

25 Slope of straight line passes through (2 , 3) and (4 , 7) is ....

A) 3 B) 2 C) 1 D) $\frac{1}{2}$

26 Slope of straight line passes through (3 , y) and (5 , – 2) is – 3 , then y = ...

A) 2 B) 4 C) 6 D) – 30

27 Slope of straight line passes through (2 , 6) and (7 , 11) is ....

A) 1 B) 2 C) 5 D) 6

28 If the Slope of straight line a X + b y + 1 = 0 is undefined , then ....

A) a = b B) a = zero C) b = zero D) a = – b

29 Relation : X – 5 = 0 is represented by a st. line whose slope is.............

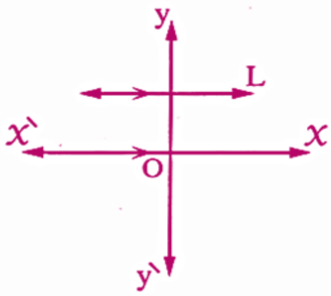
A) 0 B) – 5 C) 5 D) Undefined

30 **In the opposite figure :**

The slope of the straight line L is ....

(a) positive. (b) negative.

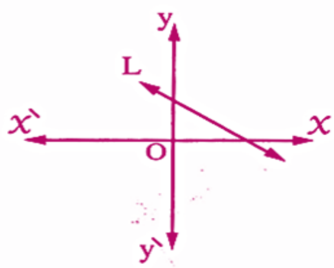
(c) zero. (d) undefined.



31 The slope of the straight line L in the opposite figure is ....

(a) positive. (b) negative.

(c) zero. (d) undefined.

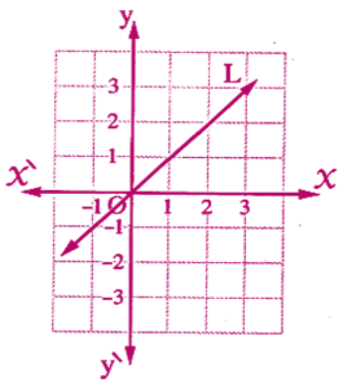


32 **In the opposite figure :**

The slope of the straight line L is ....

(a) zero. (b) undefined.

(c) 1 (d) $\frac{1}{2}$



EX. (2): Answer the following:

1	**Represent graphically the relation :** $x + y = 2$
2	**Represent the relation :** $x + 2y = 3$ graphically
3	**Represent the relation :** $2x = 5$ graphically.
4	If the straight line which represents the relation : $2x - y = a$ intersects the x-axis at the point (3 , b) , find a and b « 6 , 0 »
5	Find the value of b , where (– 3 , 2) satisfies the relation : $3x + by = 1$
6	**Find the value of :** k where (k , 2 k) satisfies the relation : $x + y = 15$
7	If (3 , a) satisfies the relation : $y - 2x = 4$, find the value of a
8	**Using the linear relations , complete the following tables :**
9	Graph the relation : $2x + 3y = 6$ If the straight line representing this relation intersects the x-axis at point A and the y-axis at point B , find the area of the triangle OAB where O is the origin point. « 3 square units »

Question 8:

(1) $4x - y = -1$

x	0	1	2	3
y	....	....	....	....

(2) $y = 5x + 15$

x	– 4	– 3	– 2
y	....	....	....

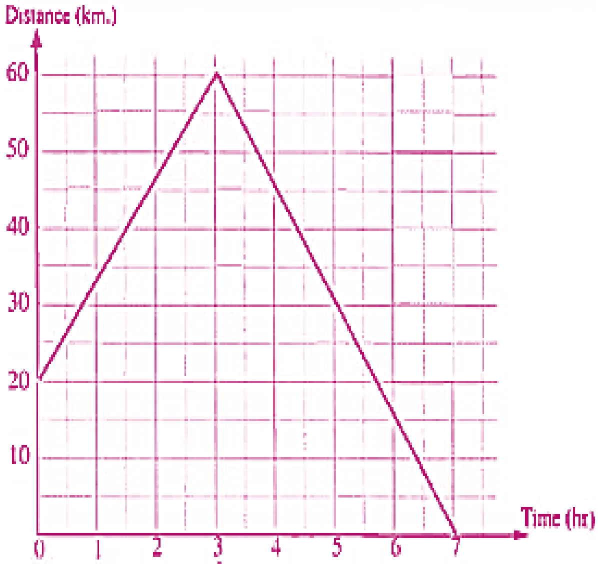
10 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »

11 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »

12 **The opposite figure represents the motion of a bicycle measured from a constant point.**
Find the regular velocity of the bicycle during :

1. The first three hours.
2. The next four hours.

Find the total distance covered by the bicycle.



« $13\frac{1}{3}$ km./hr. , 15 km./hr. , 100 km. »

13 **In each of the following , prove that the points A , B and C are collinear :**

1. A (1 , 1) , B (2 , 2) , C (–3 , –3)
2. A (4 , –3) , B (–6 , 7) , C (5 , –4)
3. A (–2 , 12) , B (2 , 4) , C (6 , –4)

Explaining

Unit 3

Lesson (1)
Collecting and Organizing Data

Example In the following table , these are the marks of 54 students in one of the classes in grade two preparatory in a school , which they took in an exam in mathematics where the full mark is 60

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	20	42	28	50
47	55	27	45	30	42	51	43	46
29	43	59	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

Solution

1 Determine the range

(it is the difference between the greatest mark and the smallest mark)

$\because$ The smallest mark is 20 and the greatest mark is 59

$\therefore$ The range = 59 – 20 = 39

2 Divide these data into a suitable number of sets of marks , say 10 disjoint sets , the length of each of them is 4 , then you obtain the following sets :

- **The first set :**
  The students who obtain 20 marks till less than 24 marks , which is written as (20 –)

- **The second set :**
  The students who obtain 24 marks till less than 28 marks , it is written as (24 –)

- **The third set :**
  The students who obtain 28 marks till less than 32 marks , it is written as (28 –) and so on till you reach the tenth set.

- **The tenth set :**
  The students who obtain 56 marks till less than 60 , it is written as (56 –)

3 Form the tally table as follows :

Sets	Tallies	Frequency
20 –	/	1
24 –	///	3
28 –	////	4
32 –	////	4
36 –	~~////~~ //	7
40 –	~~////~~ ~~////~~	10
44 –	~~////~~ ~~////~~ //	12
48 –	~~////~~ //	7
52 –	///	3
56 –	///	3
Total		54

(The tally table)

4 Omit the tallies column from the table to get the final form of the frequency table with sets. It can be written vertically or horizontally.

The following is the horizontal form of the frequency table :

Sets	20-	24-	28-	32-	36-	40-	44-	48-	52-	56-	Total
Frequency	1	3	4	4	7	10	12	7	3	3	54

From the previous table , we deduce that :

- The set that has the greatest frequency is 44 –
- The set that has the least frequency is 20 –

TRY by yourself

The following is the weights of 50 persons :

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets.

Lesson (2)

The ascending and descending cumulative frequency tables and their graphical representation

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1. The number of workers whose weekly wages are less than 60 pounds.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

• Form the ascending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

Less than 54 = 0

Less than 58 = 5 + 0 = 5

Less than 62 = 5 + 12 = 17

Less than 66 = 5 + 12 + 22 = 39

Less than 70 = 5 + 12 + 22 + 7 = 46

Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

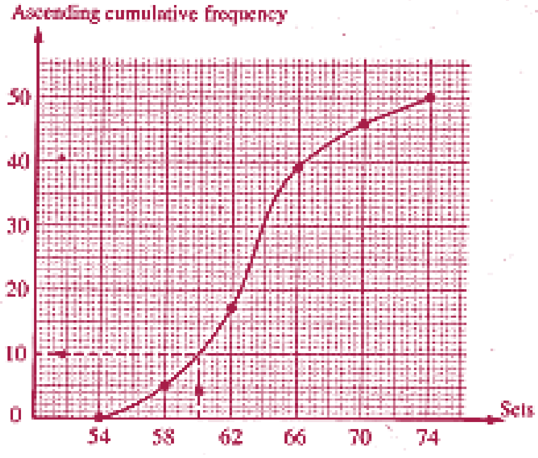
The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

Notice that : The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

1. Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.
2. Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.
3. Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.



The ascending cumulative frequency curve

• From the graph , we find that :

1. The number of workers whose weekly wages are less than 60 pounds = 10 workers.
2. The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\% = 20\%$

Example (2)

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are 60 pounds or more.

2 The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

- Form the descending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50

58 and more = 12 + 22 + 7 + 4 = 45

62 and more = 22 + 7 + 4 = 33

66 and more = 7 + 4 = 11

70 and more = 4

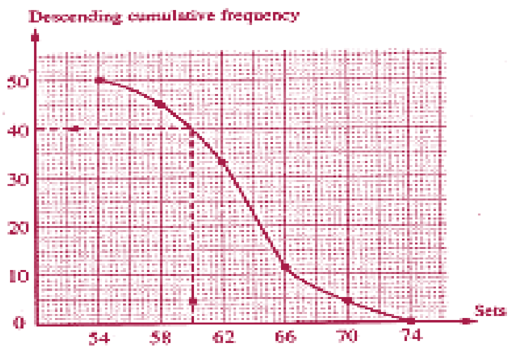
74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that : The descending cumulative frequency begins with the total frequency and ends with zero.

- To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.



The descending cumulative frequency curve

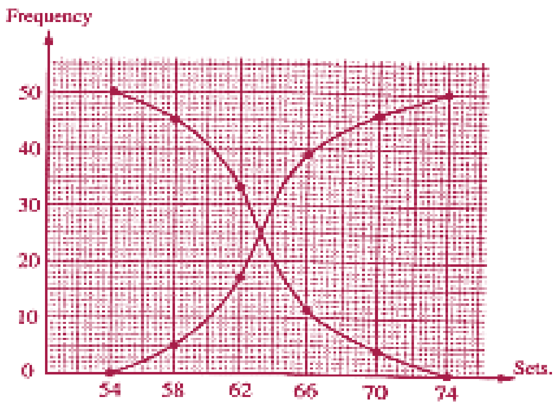
- From the graph , we find that :

1 The number of workers whose weekly wages are 60 pounds or more = 40 workers.

2 The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

Remark

we can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



Explaining

Lesson (3)

The Mean

Remember that :

To calculate the mean of a set of values, do as follows :

1. Find the sum of these values.
2. Divide this sum by the number of these values

i.e. The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that : $23 \times 5 = 115$

, the sum of marks of the 5 students $= 25 + 23 + 21 + 22 + 24 = 115$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Finding the mean of data from the frequency table with sets

Example The following table shows the distribution of the marks of 50 students in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

1 Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set $= \frac{10 + 20}{2} = 15$

, the centre of the second set $= \frac{20 + 30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \frac{50 + 60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

EX. (1): Choose the correct answer:

Exercises

1	The arithmetic mean of : 3 , 10 , 2 is …………… (a) 10 (b) 5 (c) 3 (d) 6
2	The mean of the values : 2 , 5 , 4 , 5 is …………… (a) 4 (b) 5 (c) 16 (d) 8
3	The mean of the values : 2 , 8 , 6 , 4 is …………… (a) 2 (b) 5 (c) 4 (d) 6
4	The arithmetic mean of : 3 , 7 , 28 , 52 , 10 = …………… (a) 17 (b) 19 (c) 20 (d) 27
5	The arithmetic mean of the values : 19 , 32 , 21 , 6 , 12 is …………… (a) 90 (b) 32 (c) 18 (d) 6
6	The mean of the values : 7 , 15 , 19 , 14 and 15 is ………. (a) 14 (b) 15 (c) 16 (d) 17
7	The arithmetic mean of the values : 30 , 23 , 25 , 30 , 22 is ………. (a) 22 (b) 23 (c) 24 (d) 26
8	If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 and k is 14 , then k = ………. (a) 3 (b) 6 (c) 27 (d) 84
9	If the mean of marks of 5 pupils is 20 , then the total of their marks = ………… marks. (a) 4 (b) 15 (c) 25 (d) 100
10	If the sum of 5 numbers equals 30 , then the arithmetic mean of these numbers is …………… (a) 150 (b) 6 (c) 18 (d) 72

11	The set which its lower boundary is 2 and its upper boundary is 6 , then its centre is .... (a) 2 (b) 6 (c) 4 (d) 8
12	The lowest limit of a set is 4 and the other limit is 8 , then its centre is .... (a) 2 (b) 4 (c) 6 (d) 8
13	If the lower limit of a set is 6 and the upper limit is 10 , then its centre is .... (a) 4 (b) 6 (c) 10 (d) 8
14	If the upper limit of a set is 19 and the lower limit of the same set is 11 , then the centre is .... (a) 10 (b) 15 (c) 20 (d) 30
15	If the lowest boundary of a set is 10 and the upper boundary is X and its centre is 15, then X = ... (a) 10 (b) 15 (c) 20 (d) 30
16	If the lower limit of a set is 18 and its centre is 20 , then its length is .... (a) 2 (b) 19 (c) 22 (d) 4
17	The arithmetic mean of the values : 3 – a , 5 , 1 , 4 , 2 + a equals .... (a) 1 (b) 2 (c) 3 (d) 15
18	If the arithmetic mean of the values : 9 , 6 , 5 , 14 , k is 7, then k = ... (a) 1 (b) 5 (c) 34 (d) 35
19	The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is .... (a) 1 (b) 2 (c) 3 (d) 15

EX. (2): Answer the following:

1 Find the mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	Total
Frequency	6	8	4	2	20

2 Find the arithmetic mean of the following frequency distribution :

Sets	16 –	20 –	24 –	28 –	32 –	36 –	Total
Frequency	10	15	22	25	20	8	100

3 Using the following frequency distribution to find the mean :

Sets	15 –	25 –	35 –	45 –	55 –	65 –	75 –	Total
Freq.	2	3	5	8	6	4	2	30

4 Using the following distribution , find the arithmetic mean :

Height (in cm.)	140 –	144 –	148 –	152 –	156 –	160 –	Total
Frequency	12	20	38	22	17	11	120

5 The following table shows the frequency distribution of marks of 10 students in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1. Calculate the mean of marks of students.
2. If the mark of success is 30 , calculate the number of failed students.

6 🕮 The following table shows the frequency distribution of 50 workers days-off :

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	4	5	8	k – 2	7	5	1	50

Find :

1. The value of k
2. The mean.

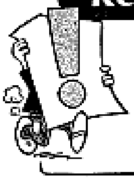
7 🕮 If the mean of the scores of a student during the first 5 months is 23.8 , what is the score of the 6^{th} month if the mean of his scores is 24 marks ?

Explaining

Lesson (4)

The Median

Remember that



The median is the middle value in a set of values after arranging it ascendingly or descendingly, such that the number of values which are less than it is equal to the number of values which are greater than it.

• To find the median of a set of values , we do as follows :

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , 23 , 30 , 42

$\boxed{\text{The median} = 23}$

If the values number is even , then

The median

$= \dfrac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , $\boxed{21 , 23}$, 24 , 27

$\boxed{\text{The median} = \dfrac{21 + 23}{2} = 22}$

Finding the median of a frequency distribution with sets graphically

Example **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

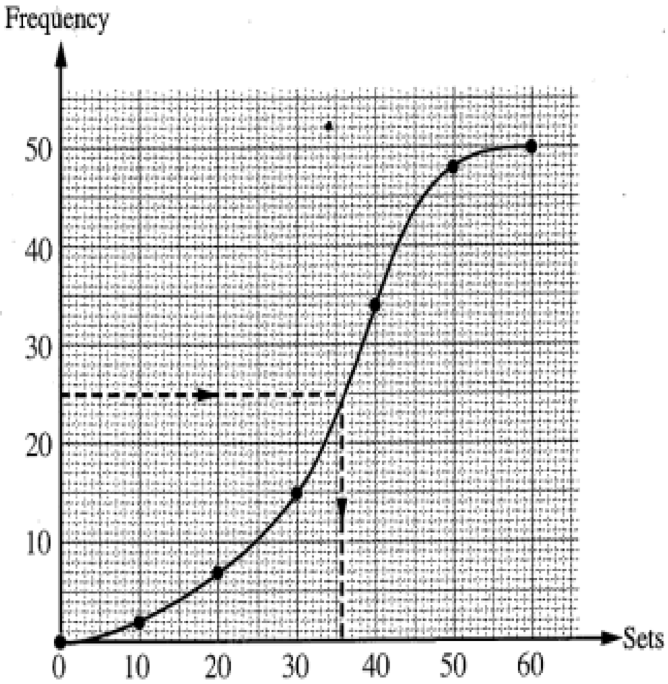
Solution

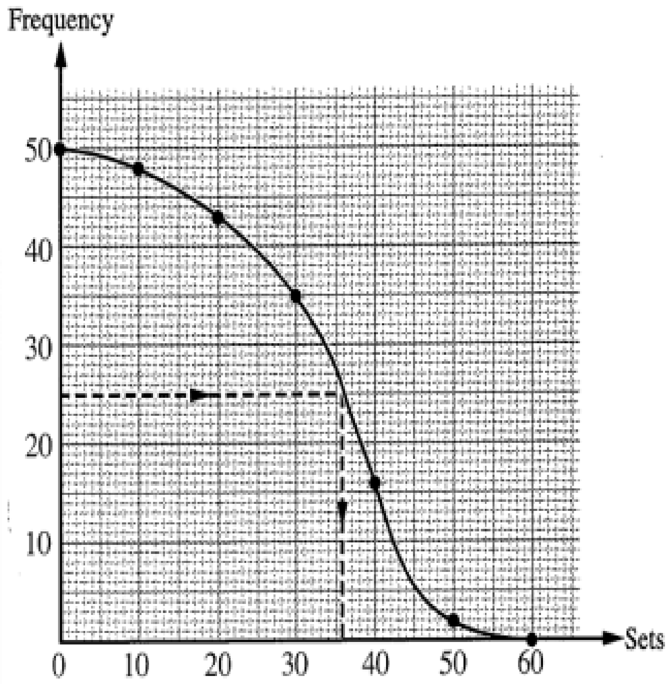
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0





$\because$ The order of the median = $\frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

EX. (1): Choose the correct answer:

Exercises

1	The order of the median of the set of values : 8 , 4 , 7 , 6 , 5 is .... (a) 7 (b) 6 (c) 3 (d) 5
2	The order of the median of the set of values : 4 , 5 , 6 , 7 and 8 is .... (a) third. (b) fourth. (c) fifth. (d) sixth.
3	If the order of the median of a set of values is the fourth , then the number of these values is .... (a) 3 (b) 5 (c) 7 (d) 9
4	If the median of the set of the values : 27 , 45 , 19 , 24 and 28 is X , then $X =$ .... (a) 24 (b) 27 (c) 28 (d) 45
5	The median of the values : 1 , 2 , 5 , 3 and 4 is .... (a) 3 (b) 4 (c) 5 (d) 2
6	The median of the values : 2 , 9 , 3 , 7 , 5 is .... (a) 5 (b) 6 (c) 7 (d) 8
7	The median of the values : 3 , 7 , 5 , 8 , 2 is .... (a) 3 (b) 5 (c) 8 (d) 7
8	The median of the values : 7 , 2 , 3 , 5 , 4 is .... (a) 3 (b) 4 (c) 5 (d) 7
9	The median for the values 3 , 9 , 7 , 4 and 5 is .... (a) 5 (b) 4 (c) 7 (d) 9
10	The median of the set of the values : 3 , 6 , 6 , 7 , 9 , 11 , 13 , 14 , 15 and 20 is .... (a) 9 (b) 10 (c) 11 (d) 20
11	The median of values : 4 , 8 , 3 , 5 , 7 , 9 is .... (a) 5 (b) 6 (c) 7 (d) 8
12	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is .... (a) 9 (b) 15 (c) 18 (d) 90

13	The median of the values : 10 , 9 , 11 , 19 , 12 is				
	(a) 9	(b) 10	(c) 11	(d) 19	
14	The median of the set of the values : 15 , 22 , 9 , 11 and 33 is				
	(a) 9	(b) 15	(c) 18	(d) 90	
15	The median of the values : 34 , 23 , 25 , 40 , 22 , 14 is				
	(a) 22	(b) 33	(c) 24	(d) 25	
16	The median of the values : 41 , 23 , 15 , 30 , 20 is				
	(a) 23	(b) 15	(c) 30	(d) 20	

EX. (2): Answer the following:

1 The following table shows the frequency distribution for the scores of 50 students in an examination :

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Find : (1) The mean of the student's score.

(2) The median. « 16.8 , 17.6 »

2 From the following frequency table with equal sets in range :

Sets	10 –	20 –	x –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

(1) Find the value of each of x and k « x = 30 , k = 15 »

(2) Graph the ascending and descending cumulative curves on one figure , then calculate the median. « 41 »

3 The following table shows the frequency distribution of weights of 20 children in kg. :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	4	7	4	2	20

Find the median weight in kg. using the ascending and descending cumulative frequency curves of this distribution. « 29 kg. »

4 The following table shows the frequency distribution of 50 workers' wages in pounds :

Sets of wages	300 –	400 –	500 –	600 –	700 –	Total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve , then find the median. « 520 pounds »

Explaining

Lesson (5)

The Mode

Remember that

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example :

The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Finding the mode for a frequency distribution with equal sets in range.

The following is an example which shows how to find the mode of a frequency distribution with sets.

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

Set of marks	10 –	20 –	30 –	40 –	50 –	**Total**
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

Solution

We can find the mode of that distribution graphically using the histogram as follows :

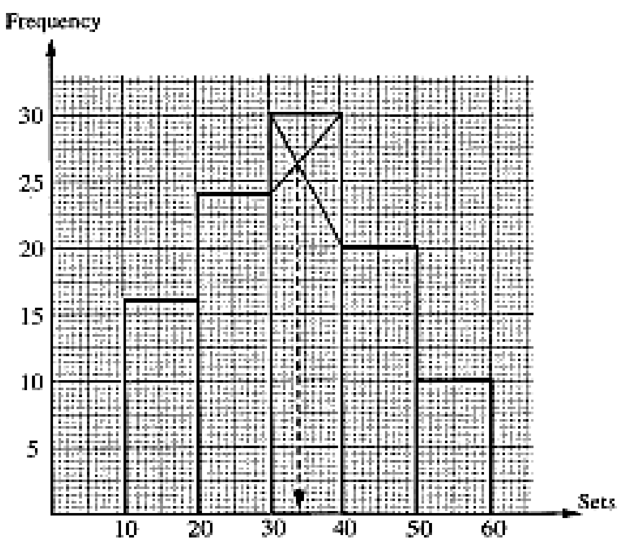
1 Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.

2 Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.

3 Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.

4 Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)

5 Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



EX. (1): Complete the following:

Exercises

1	The most common value in a set is called …………
2	The value which is the most common of a set of values is called ………………
3	The mode of a set of values is …………
4	The mode of the values : 2 , 5 , 1 , 4 , 2 is ………………
5	The mode of the values : 4 , 7 , 5 , 7 , 6 , 8 , 7 , 5 is ………………
6	The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is …………
7	The mode of the set of values : 13 , 12 , 4 , 13 is ………………
8	The mode of the set of the values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is …………
9	The mode of the values : 11 , 13 , 11 , 14 , 11 , 12 is ………………
10	The mode of the set of the values : 14 , 11 , 15 , 11 , 14 , 15 , 11 is …………
11	The mode value of : 13 , 23 , 46 , 33 , 46 , 43 , 33 , 46 , 32 is ………… cm.
12	If the mode of the set of the values : 4 , 5 , a , 3 is 4 , then a = …………
13	If the mode of the values : 3 , 6 , a , 2 , 5 is 6 , than a = ………………
14	If the mode of the set of the values : 4 , 5 , a and 3 is 3 , then a = …………
15	If the mode of the values : 5 , 7 and $x + 1$ is 7 , then x = …………
16	The mode of the values : 14 , 8 , $x + 1$, 8 , 14 is 8 , then x = ………………
17	If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then x = …………
18	If the mode of the set of the values : 15 , 9 , $x + 1$, 9 and 15 is 9 , then x = …………
19	If the mode of the set of the values : 15 , 9 , $x + 6$, 9 and 15 is 9 , then x = …………
20	If the mode of the values : 4 , 11 , 8 , and 2^{x} is 4 , then x = …………

EX. (2): Choose the correct answer:

1	The mode of the values : 3 , 5 , 3 , 6 , 3 and 8 is .... (a) 3 (b) 5 (c) 6 (d) 8
2	The mode of the sets of values : 14 , 11 , 10 , 11 , 14 , 15 , 11 is .... (a) 14 (b) 11 (c) 15 (d) 10
3	If the mode of the set of the values : 4 , 11 , 8 , $2x$ is 4 , then $x =$ .... (a) 2 (b) 4 (c) 6 (d) 8
4	The mode of the values : 15 , 9 , $x + 1$, 9 , 15 is 9 , then $x =$ .... (a) 9 (b) 14 (c) 10 (d) 8
5	The mode of 7 , 8 , 9 , $x + 2$ and 6 is 9 then $x =$ .... (a) 4 (b) 5 (c) 6 (d) 7
6	The mode of : 5 , 6 , 7 , $x + 2$ and 8 is 7 , then $x =$ .... (a) 7 (b) 6 (c) 4 (d) 5
7	If the mode of the set of values : 4 , 11 , $x + 3$, 6 is 6 , then $x =$ .... (a) 2 (b) 3 (c) 4 (d) 6
8	The mode of the set of values : 5 , 9 , 5 , $x - 2$, 9 is 9 , then $x =$ .... (a) 5 (b) 57 (c) 9 (d) 11

EX. (2): Answer the following:

1 The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :

Sets of wages in L.E.	70–	80–	90–	100–	X–	120–	130–
Number of workers	10	13	k – 4	20	16	14	11

Find : (1) The value of each of X and k « $X = 110$, k = 20 »

(2) The mode of wages in L.E. « 105 pounds »

2 The following is the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus in L.E.	20–	30–	40–	50–	60–	70–
No. of workers	10	k	22	26	20	8

(1) Calculate the value of k « 14 »

(2) Find the mean of this distribution. « 50.6 pounds »

(3) Find the mode value of the weekly bonus using the histogram. « 54 pounds »

3 The following table shows the frequency distribution for the weights of 50 students in kg. at a school :

Weight in kg.	30–	35–	40–	45–	50–	55–	Total
Number of students	7	3 k	4 k	10	8	4	50

(1) Find the value of k « 3 »

(2) Calculate the mean. « 44 kg. »

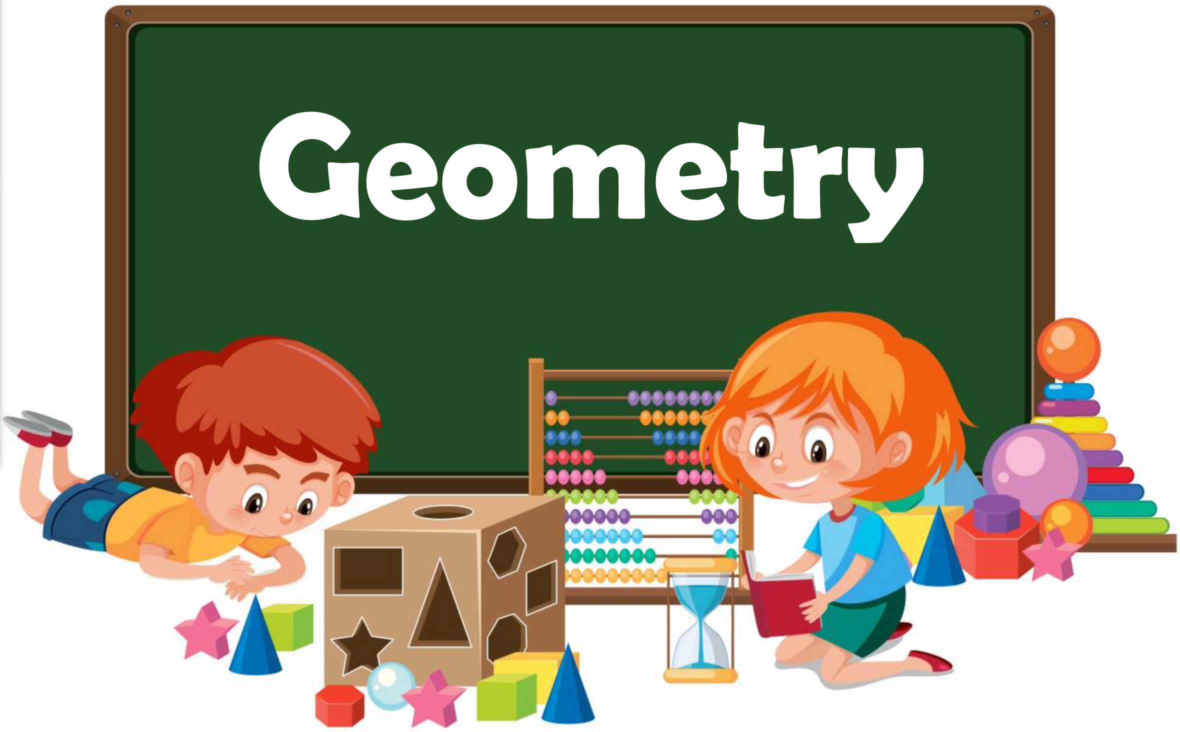
(3) Draw the ascending cumulative frequency curve.

(4) Draw the histogram and find the mode of weights. « 43 kg. »

(5) Find the median. « 43.5 kg. »

Simplest Maths

Maths-Prep2/first term (2025)



Unit 4

1. Medians of triangle.
2. Medians of triangle "Follow".
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorems.

Unit 5

6. Inequality.
7. Comparing the measures of angles in a triangle.
8. Comparing the lengths of sides in a triangle.
9. Triangle inequality.

Mr.Mohamed El-Shourbagy / 01093149109

Explaining

Unit 4

Lesson (1)
Medians of Triangle

Definition

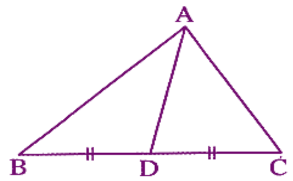
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of Δ ABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

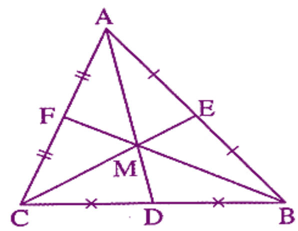
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of Δ ABC ,

and they are concurrent at M

(i.e. $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

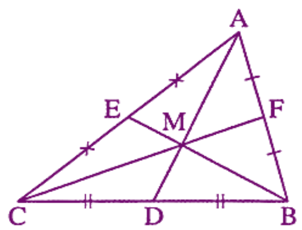
For example:

In the opposite figure :

In Δ ABC , M is the point of concurrence of its medians , then :

1 $MD = \frac{1}{2} AM$ If AM = 6 cm. , then MD = 3 cm.

2 CM = 2 FM If FM = 4 cm. , then CM = 8 cm.



Remark

The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.

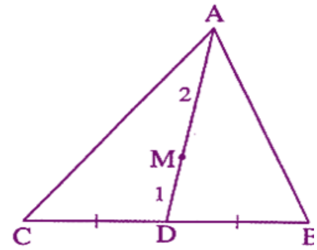
Fact

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC and M $\in \overline{AD}$ such that AM = 2 MD ,

then M is the point of intersection of the medians of Δ ABC



Example:

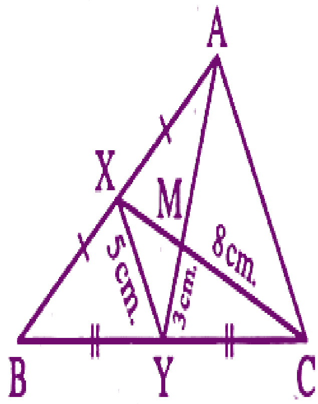
In the opposite figure :

ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$, $\overline{XC} \cap \overline{AY} = \{M\}$

, XY = 5 cm. , CM = 8 cm. , YM = 3 cm.

Find the perimeter of : Δ MAC



Solution

In Δ ABC

$\because$ Y is a midpoint of $\overline{BC}$

$\because$ X is a midpoint of $\overline{AB}$

$\therefore$ AC = 2 XY = 2 × 5 = 10 cm

$\because$ Y is a midpoint of $\overline{BC}$

$\therefore$ $\overline{AY}$ is a median In Δ ABC

$\because$ X is a midpoint of $\overline{AB}$

$\therefore$ $\overline{CX}$ is a median In Δ ABC

$\therefore$ M is the intersection point of medians In Δ ABC

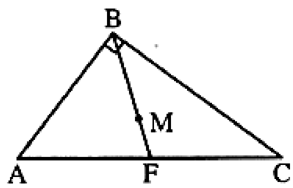
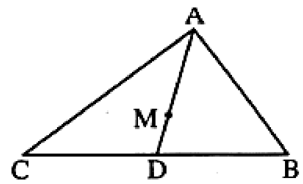
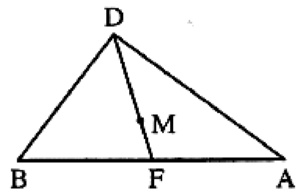
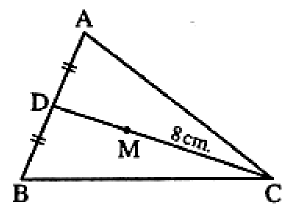
$\therefore$ AM = 2 MY = 2 × 3 = 6 cm

The perimeter of Δ MAC =

6 + 8 + 10 = 24 cm

EX. (1): Complete the following:

Exercises

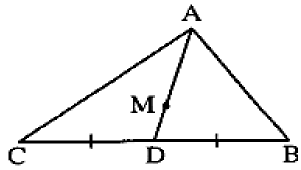
1	In ΔABC : if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ……….
2	The medians of the triangle are ……….
3	The medians of the triangle intersect at ……….
4	The point of intersection of the medians of a triangle divides each median in the ratio ………. from the vertex.
5	The points of concurrence of the medians of the triangle divides each median in the ratio ………. : ………. from the base.
6	The point of intersection of the medians of the triangle divides each of them by the ratio 1 : 2 from ……….
7	The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of ……………
8	**In the opposite figure :** If M is intersection point of medians and $m(\angle B) = 90°$, MF = 1.5 cm. , then the length of $\overline{AC}$ = ………. 
9	**In the opposite figure :** If M is the point of intersection of the medians of ΔABC , then AM = ………. AD 
10	**In the opposite figure :** If : MF = 2 cm., then DF = ………. 
11	**In the opposite figure :** In ΔABC , M is the point of concurrence of the medians , MC = 8 cm. , then DM = ………. cm. 

EX. (2): Choose the correct answer:

1	The medians of the triangle intersect at .... point. (a) 1 (b) 2 (c) 3 (d) 4
2	The right-angled triangle has .... medians. (a) 0 (b) 1 (c) 2 (d) 3
3	The number of medians in the right-angled triangle = ... (a) 3 (b) 2 (c) 1 (d) 0
4	The point of intersection of the medians in the triangle divides each of them by the ratio .... from the vertex. (a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2
5	The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base. (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1
6	If $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ... AD (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
7	$\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then AM = ... MD (a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$
8	If $\overline{XE}$ is a median in Δ XYZ , M is the point of intersection of its medians , then EM = ... XE (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
9	In Δ ABC : If AD = 6 cm. is a median and M is a point of concurrent , then MA = ... cm. (a) 6 cm. (b) 3 cm. (c) 2 cm. (d) 4 cm.

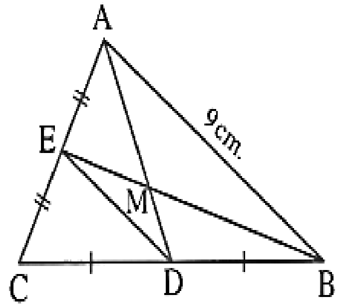
10 If $\overline{AD}$ is a median of ΔABC , M is the point of intersection of its medians and AM = 6 cm. , then AD = ………
(a) 12 cm. (b) 6 cm. (c) 18 cm. (d) 9 cm.

11 **In the opposite figure :**
$\overline{AD}$ is a median in ΔABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = …………… cm.
(a) 2 (b) 4 (c) 6 (d) 8



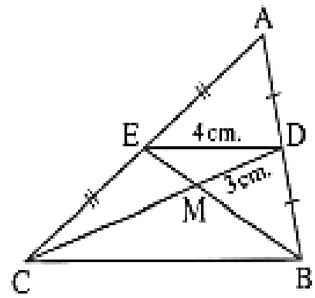
EX. (3): Answer the following:

1 **In the opposite figure :**
ABC is a triangle in which D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ and $\overline{AD} \cap \overline{BE} = \{M\}$
If AD = 6 cm. and AB = BE = 9 cm.
Calculate : The perimeter of Δ MDE



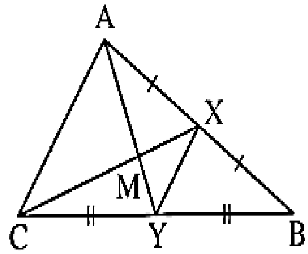
« 9.5 cm. »

2 **In the opposite figure :**
If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ and $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm. , DM = 3 cm. and BE = 6 cm.
Find : The perimeter of Δ BMC



« 18 cm. »

3 🕮 **In the opposite figure :**
ABC is a triangle , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$, XY = 5 cm. and $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm. , YM = 3 cm. **Find :**



$\boxed{1}$ The perimeter of Δ MXY $\boxed{2}$ The perimeter of Δ MAC « 12 cm. , 24 cm. »

4

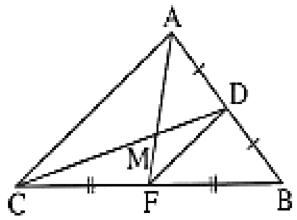
In the opposite figure :

$\overline{AF}$ and $\overline{CD}$ are two medians in ΔABC ,

$\overline{AF} \cap \overline{CD} = \{M\}$

If the perimeter of $\Delta AMC = 36$ cm.

Find : The perimeter of ΔMFD



« 18 cm. »

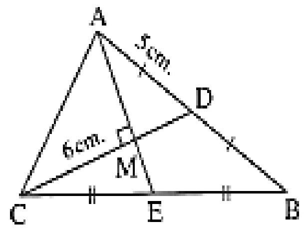
5

In the opposite figure :

M is the point of concurrence of the medians of ΔABC , $\overline{AM} \perp \overline{CD}$

, MC = 6 cm. , AD = 5 cm.

Find : The length of $\overline{ME}$



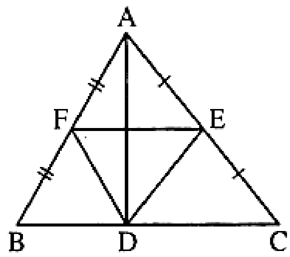
« 2 cm. »

6

In the opposite figure :

ABC is a triangle , E , F are the midpoints of $\overline{AC}$ and $\overline{AB}$ respectively , $\overline{AD} \perp \overline{BC}$, AC = 18 cm. ,

BC = 20 cm. , AB = 16 cm.



Complete :

DF = ········ cm. , DE = ········ cm. , FE = ········ cm. , perimeter of ΔDEF = ········ cm.

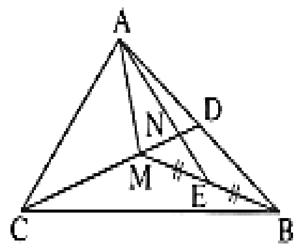
7

In the opposite figure :

$M \in \overline{CD}$, M is the point of concurrence of the medians of ΔABC , $N \in \overline{DM}$ where $ND = (X - 1)$ cm.

, $MN = (X + 3)$ cm. , $\overrightarrow{AN}$ is drawn to intersect $\overline{BM}$ at E which is the midpoint of $\overline{BM}$

Find : The length of $\overline{MC}$



« 24 cm. »

Explaining

Lesson (2)

Medians of Triangle (follow)

Theorem 3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

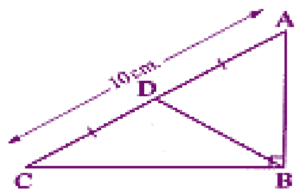
For example:

In the opposite figure :

ΔABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. ,

then DB = 5 cm.



The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.

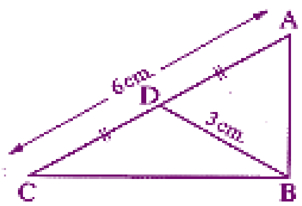
For example:

In the opposite figure :

If $\overline{BD}$ is a median in ΔABC ,

BD = 3 cm. and AC = 6 cm. ,

then m ($\angle ABC$) = 90° "because BD = $\frac{1}{2}$ AC "



Corollary

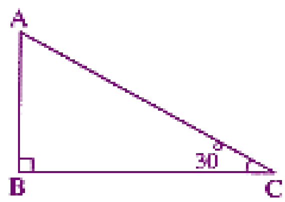
The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure :

If ΔABC is right-angled at B and

m ($\angle C$) = 30° , then AB = $\frac{1}{2}$ AC



For example:

If AC = 20 cm. , then AB = 10 cm.

Remark

The right-angled triangle whose measures of angles are 30° , 60° and 90° is called thirty and sixty triangle.

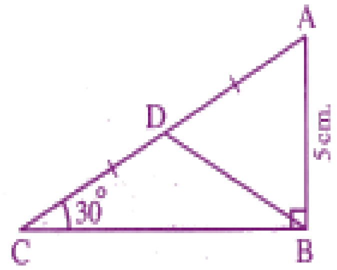
Examples

1 **In the opposite figure :**

m (∠ B) = 90° and m (∠ C) = 30°

, AB = 5 cm,

Find the length of : $\overline{AC}$ and $\overline{BD}$



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ m (∠C) = 30°

∴ AC = 2 AB = 2 × 5 = 10 cm

(First Req.)

∵ D is a midpoint of $\overline{AC}$

∴ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm

(Second Req.)

2 **In the opposite figure :**

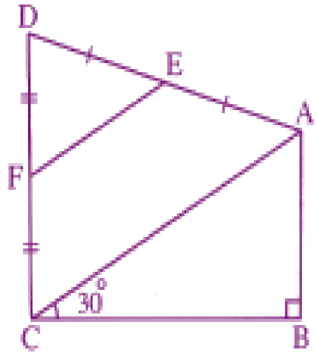
m (∠ B) = 90°

, m (∠ ACB) = 30°

, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

Prove that : AB = EF



Solution

In Δ ABC

∵m (∠B) = 90°

∵m (∠C) = 30°

∴ BD = $\frac{1}{2}$ AC - - - - - - - (1)

In Δ ADC

∵ E is a midpoint of $\overline{AD}$

∵ F is a midpoint of $\overline{DC}$

∴ EF = $\frac{1}{2}$ AC - - - - - - - (2)

From (1) ad (2)

∴ AB = EF

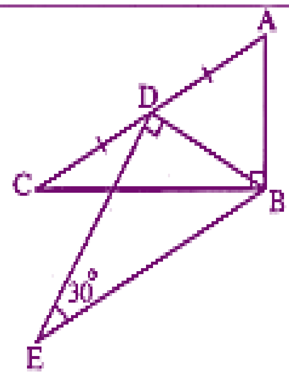
3 In the opposite figure :

$m(\angle ABC) = m(\angle BDE) = 90°$

, $m(\angle E) = 30°$

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



Solution

In Δ ABC

$\because m(\angle B) = 90°$

$\because$ D is a midpoint of $\overline{AC}$

In Δ DBE

$\because m(\angle BDE) = 90°$

$\because m(\angle E) = 30°$

$\therefore \overline{BD}$ is a median

$\therefore BD = \frac{1}{2} AC$ ------(1)

$\therefore BD = \frac{1}{2} BE$ ------(2)

From (1) ad (2)

$\therefore AC = BE$

4 In the opposite figure :

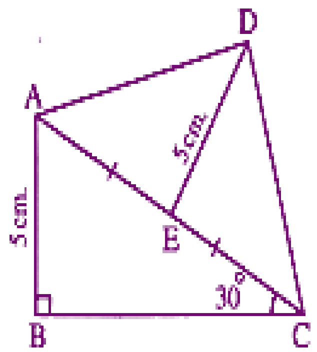
ABC is a right-angled triangle at B

, $m(\angle ACB) = 30°$, AB = 5 cm.

, E is the midpoint of $\overline{AC}$

If DE = 5 cm.

Prove that : $m(\angle ADC) = 90°$



Solution

In Δ ABC

$\because m(\angle B) = 90°$

$\because m(\angle BCA) = 30°$

$\therefore AB = \frac{1}{2} AC$

$\because AB = 5$ cm

$AC = 5 \times 2 = 10$ cm

In Δ ADC

$\because$ E is a midpoint of $\overline{AC}$

$\therefore \overline{ED}$ is a median

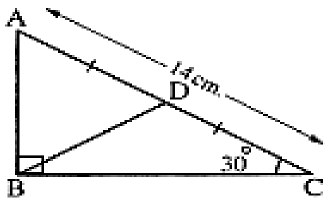
$\because DE = 5$ cm

$\therefore DE = \frac{1}{2} AC$

$\therefore m(\angle ADC) = 90°$

Exercises

EX. (1): Complete the following:

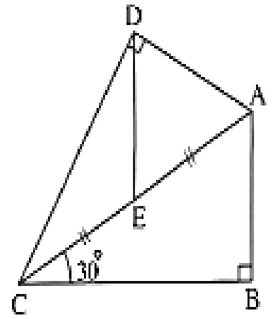
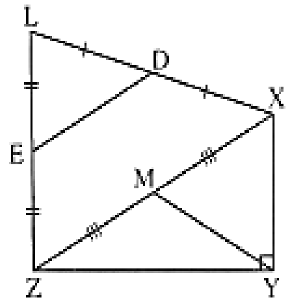
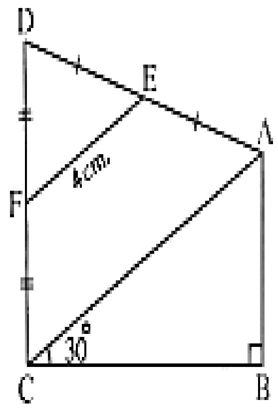
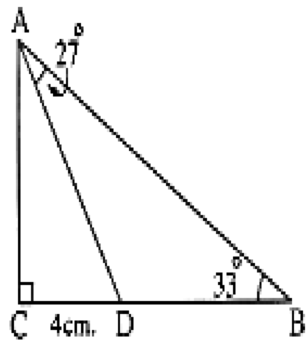
1	In the right-angled triangle the length of the median from the vertex of the right angle equal ········ the length of the hypotenuse.
2	In the right-angled triangle , the length of the median from the vertex of the right angle equals ········
3	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then ········
4	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ········ the length of the hypotenuse.
5	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ········
6	The length of the hypotenuse on the right-angled triangle equals ········ the length of a side opposite to the angle of measure 30°
7	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ········ cm.
8	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ········ cm.
9	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ·············· cm.
10	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ········ AC
11	If ABC : Is a right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ C) = ········
12	If ABC is a right-angled triangle at B and AB = $\frac{1}{2}$ AC , then m (∠ A) = ········
13	ABC is a right-angled triangle at B , if AC = 2 BC , then m (∠ C) = ··············°
14	In the opposite figure : The perimeter of Δ ABD = ········ cm. 

EX. (2): Choose the correct answer:

1	The length of the hypotenous of the right-angled triangle = ……… the length of the median which drawn from the vertex of the right-angle. (a) half (b) twice (c) third (d) quarter
2	The length of the median drawn from the vertex of right angle in the right-angled triangle = ……… the length of the hypotenuse of the triangle. (a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
3	In the right-angled triangle , the length of the median from the vertex of the right angle equal ……… the length of the hypotenuse. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2
4	In the right-angled triangle , the length of the median from the vertex of the right angle equals ……… the length of hypotenuse. (a) half (b) twice (c) third (d) forth
5	If Δ ABC is a right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the medians drawn from B is ……… cm. (a) 10 (b) 8 (c) 6 (d) 5
6	In Δ ABC which is right at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ……… (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.
7	In Δ ABC , m ($\angle$ B) = 90° , AC = 12 cm. and $\overline{BD}$ is a median in Δ ABC , then BD = ……… cm. (a) 12 (b) 6 (c) 24 (d) 10
8	The length of the side opposite to the angle of measure 30° in the right-angled ……… the length of the hypotenuse. (a) twice (b) half (c) square (d) equals
9	Triangle ABC : If m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , then BC = ……… (a) $\frac{1}{2}$ AB (b) $\frac{1}{2}$ AC (c) 2 AB (d) 2 AC
10	In Δ ABC if : m ($\angle$ B) = 90° and m ($\angle$ A) = 60° , then AC = ……… AB (a) 2 (b) = (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
11	Δ ABC : if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then AC = ……… (a) $\frac{1}{2}$ BC (b) 2 BC (c) 2 AB (d) BC

12	In Δ ABC : m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , AC = 10 cm. , then BC = ……… cm. (a) 20 (b) 15 (c) 10 (d) 5
13	In Δ XYZ , if m ($\angle$ Y) = 90° , m ($\angle$ X) = 30° and XZ = 20 cm. , then ZY = …………… cm. (a) 5 (b) 8 (c) 20 (d) 10
14	In the rectangle ACBD , if AC = 10 cm. , then BD = ……… (a) 5 (b) 10 (c) 15 (d) 20

EX. (3): Answer the following:

1	**In the opposite figure :** $m(\angle ABC) = m(\angle ADC) = 90°$, $m(\angle ACB) = 30°$ and E is the midpoint of $\overline{AC}$ **Prove that :** AB = DE 
2	**In the opposite figure :** $m(\angle XYZ) = 90°$, D is the midpoint of $\overline{XL}$, E is the midpoint of $\overline{ZL}$ and M is the midpoint of $\overline{XZ}$ **Prove that :** DE = YM 
3	**In the opposite figure :** ABCD is a quadrilateral in which $m(\angle B) = 90°$, E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$, $m(\angle ACB) = 30°$ and EF = 4 cm. **Find by proof :** The length of $\overline{AB}$ « 4 cm. » 
4	**In the opposite figure :** ABC is a triangle in which $m(\angle B) = 33°$, $m(\angle C) = 90°$, D ∈ $\overline{BC}$ where CD = 4 cm. , $m(\angle BAD) = 27°$ **Find :** The length of $\overline{AD}$ « 8 cm. » 

5

In the opposite figure :

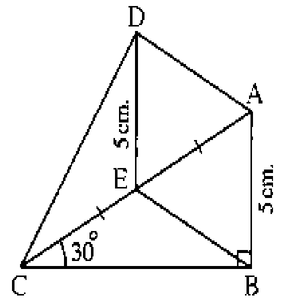
ABC is a right-angled triangle at B ,

m ($\angle$ ACB) = 30° , AB = 5 cm. and

E is the midpoint of $\overline{AC}$

If DE = 5 cm. ,

prove that : m ($\angle$ ADC) = 90°



6

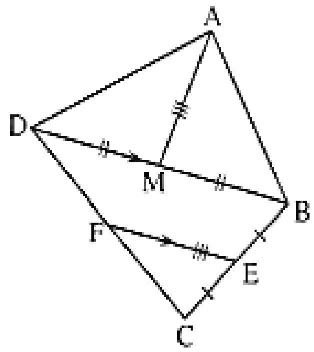
In the opposite figure :

ABD is a triangle , M is the midpoint of $\overline{BD}$,

E is the midpoint of $\overline{BC}$,

F $\in$ $\overline{CD}$, $\overline{EF}$ // $\overline{BD}$ and AM = EF

Prove that : m ($\angle$ BAD) = 90°



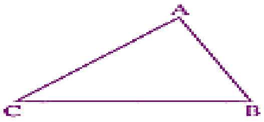
Explaining

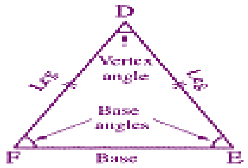
Lesson 3

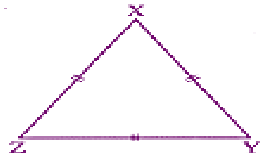
The Isosceles Triangle

Triangles are classified according to the lengths of their sides into three types which are :

1. Scalene triangle.
2. Isosceles triangle. (two sides are congruent).
3. Equilateral triangle. (three sides are congruent).







And in the following we will study the relations between the angles in the isosceles triangle and the equilateral triangle.

The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.

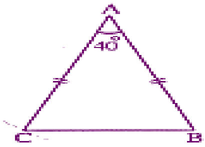
For example:

In the opposite figure :

If ABC is a triangle in which :

$AB = AC$, $m(\angle A) = 40°$,

then $m(\angle B) = m(\angle C) = \frac{180° - 40°}{2} = 70°$



Remarks

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

Corollary

If the triangle is equilateral , then it is equiangular where each angle measure is 60°

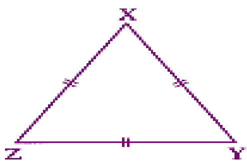
For example:

In the opposite figure :

If XYZ is a triangle in which

$XY = YZ = ZX$,

then $m(\angle X) = m(\angle Y) = m(\angle Z) = 60°$



Examples

(1) In the opposite figure :

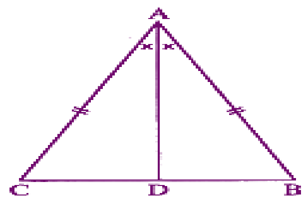
In Δ ABC :

$AB = AC$, $\overrightarrow{AD}$ bisects $\angle BAC$

and $BD = 3$ cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



Solution

In Δ ABC
$\because AB = AC$
$\because AD$ bisects $\angle BAC$
$\therefore AD \perp = BC$ (First Req.)

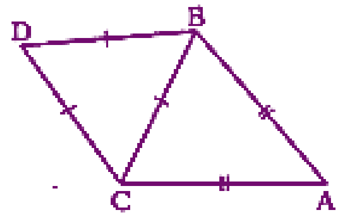
$\therefore$ D is a midpoint of BC
$\because BD = 3$ cm
$\therefore CD = BC = 3$ cm
$\therefore CB = 3 \times 2 = 6$ cm (Second Req.)

(2) In the opposite figure :

$m(\angle A) = 50°$, $AB = AC$

and ΔDBC is an equilateral.

Find : $m(\angle ABD)$



Solution

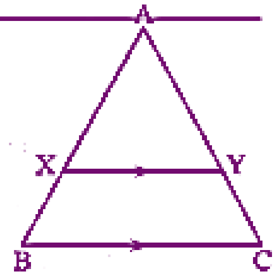
In ΔABC
$\because AB = AC$
$\because m(\angle A) = 50°$
$\therefore m(\angle ABC) = m(\angle ACB)$
$\therefore m(\angle ABC) = (180 - 50) \div 2 = 65°$
$\because \Delta DBC$ is a equilateral
$\therefore m(\angle DBC) = 60$
$\therefore m(\angle ABD) = 65 + 60 = 125°$

(3) In the opposite figure :

If $AB = AC$,

$\overline{XY} \parallel \overline{BC}$

Prove that : ΔAXY is an isosceles



Solution

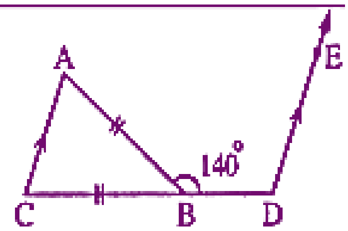
In ΔABC
$\because AB = AC$
$\therefore m(\angle B) = m(\angle C)$
$\because XY \parallel BC$, AC & AB are transversals
$\therefore m(\angle B) = m(\angle AXY)$
Corresponding
$\therefore m(\angle C) = m(\angle AYX)$
Corresponding
In ΔAXY
$\therefore m(\angle AXY) = m(\angle AYX)$
$\therefore AX = AY$
ΔAXY is an isosceles

(4) In the opposite figure :

$\overline{CA} \parallel \overrightarrow{DE}$, $m(\angle ABD) = 140°$

$AB = BC$

Find : $m(\angle EDB)$



Solution

In ΔABC $\because AB = BC$
$\therefore m\angle A = m\angle C$
$\because m\angle ABD = m\angle A + m\angle C$
(Exterior)
$\because m\angle ABD = 140°$
$\therefore m\angle A = m\angle C = 140 \div 2 = 70°$
$\because AC \parallel DE$, CD is a transversal
$\therefore m(\angle C) + m(\angle D) = 180$
(Interior)
$\therefore m(\angle D) = 180 - 70 = 110$

Exercises

EX. (1): Complete the following:

1	The two base angles in an isosceles triangle are ··········
2	Δ ABC , AB = AC , m ($\angle$ C) = 70° , then m ($\angle$ A) = ··········
3	In the Δ ABC : AB = AC , m ($\angle$ A) = 70° , then m ($\angle$ C) = ··········°
4	The Δ ABC is an isosceles and right-angled triangle if m ($\angle$ B) = 90° , then m ($\angle$ A) = m ($\angle$ C) = ················°
5	In Δ ABC , if AB = AC and m ($\angle$ A) = 80° , then m ($\angle$ B) = m ($\angle$ ················) = ·········°
6	In Δ ABC : if AB = AC , m ($\angle$ B) = 60° , then the triangle is an ··········
7	In Δ ABC : If AB = AC and m ($\angle$ A) = 2 m ($\angle$ C) , then m ($\angle$ B) = ·········· °
8	The triangle whose side lengths 3 cm. , (X + 1) , and 6 cm. become isosceles triangle when X = ··········

EX. (2): Choose the correct answer:

1	In any isosceles triangle , the type of the base angles is ………. (a) acute. (b) right. (c) obtuse. (d) reflex.
2	The base angles of the isosceles triangle are …………… (a) congruent. (b) alternate. (c) corresponding. (d) supplementary.
3	If measure of one of the two base angles of the isosceles triangle equals 40° then the measure of the vertex angle = ……………° (a) 40 (b) 100 (c) 80 (d) 50
4	In Δ ABC : AB = AC , m (∠ B) = 50° , then m (∠ A) = ……………° (a) 65 (b) 80 (c) 50 (d) 100
5	An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle = ………. (a) 50° (b) 60° (c) 70° (d) 80°
6	In the isosceles triangle , if the measure of one of the two base angle is 70° , then the measure of its vertex angle is ………. (a) 70° (b) 110° (c) 20° (d) 40°
7	The measure of one angle of the two base angles of the isosceles = 75° , then the measure of the vertex angle = ………. (a) 50° (b) 75° (c) 30° (d) 105°
8	In a triangle ABC : If AB = AC and m (∠ A) = 40° , then m (∠ C) = ………. (a) 40° (b) 70° (c) 140° (d) 50°
9	In Δ ABC , AB = AC , m (∠ A) = 50° , then m (∠ B) = ………. (a) 50° (b) 65° (c) 130° (d) 100°
10	If the measure of an angle of the isosceles triangle is 100° , then the measure of one of the other angles = …….. (a) 50° (b) 80° (c) 40° (d) 100°
11	Δ XYZ is an isosceles triangle in which m (∠ X) = 100° , then m (∠ Y) = ……….° (a) 100 (b) 80 (c) 60 (d) 40
12	ABC is a triangle in which AB = AC and m (∠ A) = 110°, then m (∠ B) = ………. (a) 70° (b) 55° (c) 35° (d) 110°
13	If the measure of an angle of the isosceles triangles is 120° , then the measure of one of the other angles = ………. (a) 60° (b) 30° (c) 40° (d) 45°
14	ABC is isosceles triangle m (∠ C) = 130° , then m (∠ B) = ……………° (a) 130 (b) 50 (c) 25 (d) 60

15 The triangle whose sides lengths are 2 cm. , $(X + 1)$ cm and 5 cm. becomes an isosceles triangle when $X =$ .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

16 The triangle whose sides lengths are 3 cm. , $(X + 5)$ and 9 becomes an isosceles if $X =$ .... cm.

(a) 3 (b) 4 (c) 5 (d) 6

17 Triangle whose sides lengths are 2 cm. , $(X - 2)$ cm. , 5 cm. becomes isosceles triangle when $X =$ .... cm.

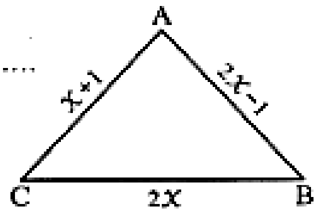
(a) 3 (b) 4 (c) 5 (d) 7

18 **In the opposite figure :**

ABC is a triangle in which : m (∠ B) = m (∠ C) , then $X =$ ....

(a) 1 (b) 2

(c) 3 (d) 4

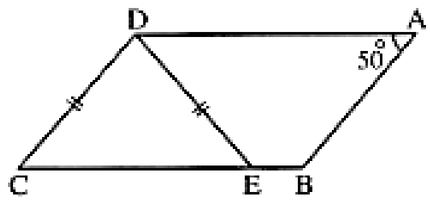


19 ABCD is a parallelogram :

DE = DC , m (∠ A) = 50° , then m (∠ EDC) = ...

(a) 50° (b) 60°

(c) 70° (d) 80°



EX. (3): Answer the following:

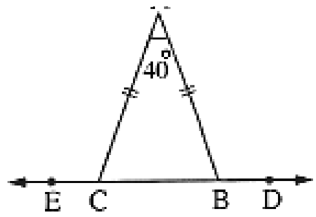
1 **In the opposite figure :**

ABC is an isosceles triangle in which AB = AC ,

m (∠ A) = 40° and D ∈ $\overrightarrow{CB}$, E ∈ $\overrightarrow{BC}$

1 **Find :** m (∠ ABC)

2 **Prove that :** ∠ ABD ≡ ∠ ACE



« 70° »

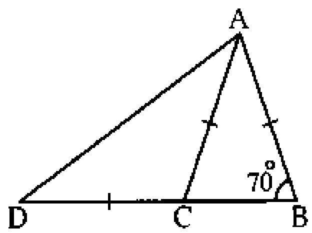
2 **In the opposite figure :**

AB = AC = CD

and m (∠ B) = 70°

Find by proof :

m (∠ BAD)



« 75° »

3

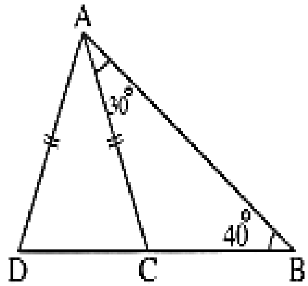
In the opposite figure :

m (∠ B) = 40° , m (∠ BAC) = 30°

and AC = AD

Find by proof :

1 m (∠ D) 2 m (∠ CAD)



« 70° , 40° »

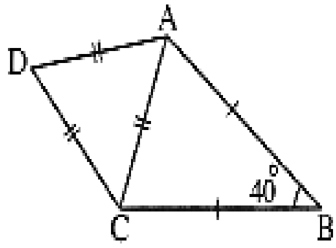
4

In the opposite figure :

AD = DC = AC , AB = BC

and m (∠ ABC) = 40°

Find : m (∠ BAD)



« 130° »

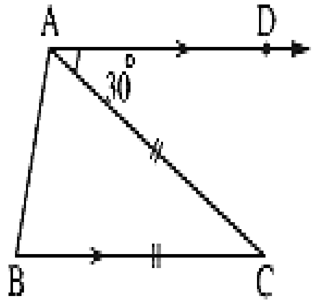
5

In the opposite figure :

ABC is a triangle in which AC = BC ,

$\overrightarrow{AD}$ // $\overline{BC}$ and m (∠ DAC) = 30°

Find : The measures of the angles of Δ ABC



« 30° , 75° , 75° »

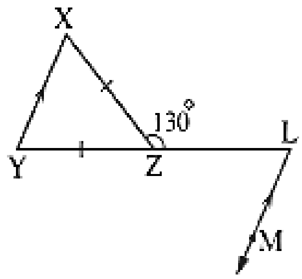
6

In the opposite figure :

Z ∈ $\overline{LY}$, XZ = YZ , m (∠ LZX) = 130°

and $\overrightarrow{LM}$ // $\overline{XY}$

Find : m (∠ MLY)



« 65° »

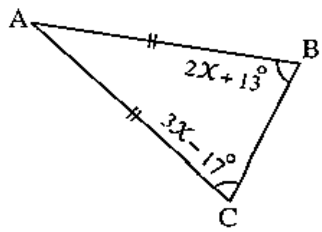
7

In the opposite figure :

AB = AC , m (∠ B) = 2 x + 13°

and m (∠ C) = 3 x – 17°

Find : The measures of the angles of Δ ABC



Explaining

Lesson (4)

Converse of the isosceles triangle theorem

To prove it is Isosceles Triangle :

Theorem 2

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Remark

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

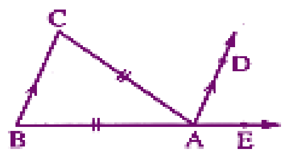
Examples on Part (1) : Isosceles Triangle

5] **In the opposite figure :**

$AB = AC$,

$\overrightarrow{AD} \parallel \overline{BC}$

Prove that : $\overrightarrow{AD}$ bisects $\angle CAE$



Solution

In ΔABC

$\because AB = AC$

$\therefore m\angle B = m\angle C$

$\because AC \parallel DE$, AB & AC are transversals

$\therefore m\angle B = m\angle DAE$ (Corresponding)

$\therefore m\angle C = m\angle CAD$ (Alternate)

$\therefore m\angle DAE = m\angle CAD$

AD bisects $m\angle CAE$

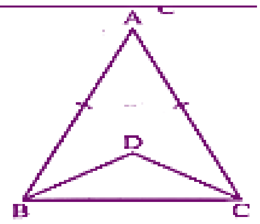
6 **In the opposite figure :**

ABC is a triangle in which $AB = AC$,

$\overrightarrow{BD}$ bisects $\angle ABC$, $\overrightarrow{CD}$ bisects $\angle ACB$

Prove that :

ΔDBC is an isosceles triangle.



Solution

In ΔABC

$\because AB = AC$

$\therefore m\angle B = m\angle C$

$\because$ BD bisects $m\angle ABC$

$\because$ CD bisects $m\angle ACB$

$\therefore m\angle DBC = \frac{1}{2} m\angle ABC$

$\therefore m\angle DCB = \frac{1}{2} m\angle ACB$

$\therefore m\angle DBC = m\angle DCB$

$\therefore \Delta DBC$ is an isosceles

EX. (1): Complete the following:

Exercises

1	If angles of any triangle are equal in measures , then the triangle is ………
2	If the angles of a triangle are congruent , then the triangle is ……………
3	The measure of the exterior angle of equilateral triangle = ………°
4	The measure of any exterior angle of the triangle is greater than ………
5	If the measure of one of the angles of the right-angled triangle is 45° , then the triangle is ……………
6	In an isosceles triangle , if any angle has a measure of 60° , the triangle is ………
7	In Δ ABC if : $\overline{AB} \perp \overline{BC}$ and AB = BC , then m ($\angle$ A) = ………°

EX. (2): Choose the correct answer:

1	The measure of exterior angle of an equilateral triangle = ……… (a) 30° (b) 60° (c) 120° (d) 180°
2	In Δ XYZ : if XY = XZ , then the exterior angle at the vertex Z is ……… (a) acute. (b) obtuse. (c) right. (d) reflex.
3	In Δ ABC : if AB = AC and m ($\angle$ A) = 60° , if its perimeter is 18 cm. , then BC = ……… cm. (a) 18 (b) 6 (c) 3 (d) 60
4	Δ ABC , AB = AC , D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is ……… (a) median. (b) altitude. (c) bisector of the vertex angle. (d) all the previous.

EX. (3): Answer the following:

1

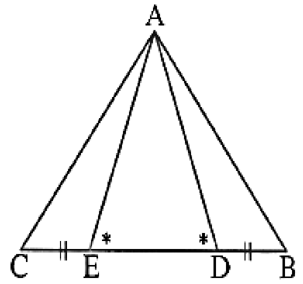
In the opposite figure :

$\angle ADE \equiv \angle AED$

, B , D , E , C are collinear

and BD = CE

Prove that : Δ ABC is an isosceles triangle.



2

ABC is a triangle in which AB = AC , $\overrightarrow{BD}$ bisects $\angle$ ABC and $\overrightarrow{CD}$ bisects $\angle$ ACB

Prove that : Δ DBC is an isosceles triangle.

3

ABC is a triangle in which D $\in \overline{AB}$ and E $\in \overline{BC}$ such that BD = BE

So if $\overline{DE}$ // $\overline{AC}$, **prove that :** AB = BC

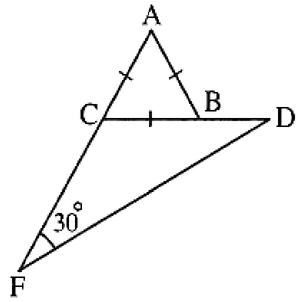
4

In the opposite figure :

ABC is an equilateral triangle , F $\in \overrightarrow{AC}$,

D $\in \overrightarrow{CB}$ and m ($\angle$ DFC) = 30°

Prove that : Δ DCF is an isosceles triangle.



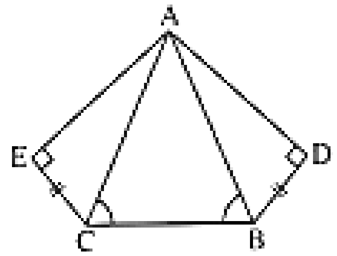
5

In the opposite figure :

BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ D) = m ($\angle$ E) = 90°

Prove that : m ($\angle$ DAB) = m ($\angle$ CAE)



6

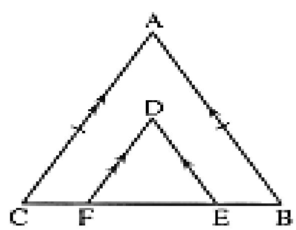
In the opposite figure :

AB = AC , $\overline{DE}$ // $\overline{AB}$ and $\overline{DF}$ // $\overline{AC}$

Prove that :

[1] DE = DF

[2] m ($\angle$ BAC) = m ($\angle$ EDF)



Explaining

Lesson (5)

Corollaries of the isosceles triangle theorem

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

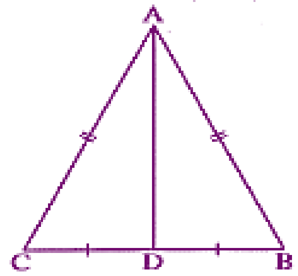
ABC is a triangle in which AB = AC and

$\overline{AD}$ is a median , then :

1 $\overrightarrow{AD}$ bisects ∠ BAC

i.e. m (∠ BAD) = m (∠ CAD)

2 $\overline{AD} \perp \overline{BC}$



Isosceles Triangle : Vertex Bisector

Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

n the opposite figure :

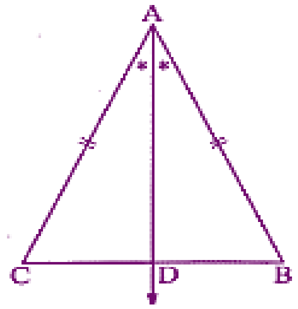
ABC is a triangle in which AB = AC and

$\overrightarrow{AD}$ bisects ∠ BAC , then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 $\overline{AD} \perp \overline{BC}$



Isosceles Triangle : Perpendicular

Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

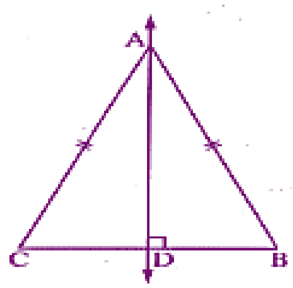
ABC is a triangle in which AB = AC and

$\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 m (∠ BAD) = m (∠ CAD)



Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

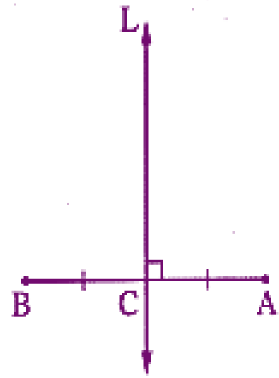
Axis of symmetry of line segment (1)

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line $L \perp \overline{AB}$ and $C \in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$



Axis of symmetry of line segment (2)

Property

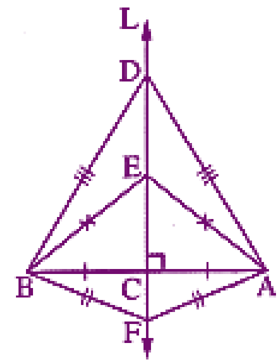
Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of $\overline{AB}$,

$D \in L$, $E \in L$ and $F \in L$, then

$DA = DB$, $EA = EB$ and $FA = FB$

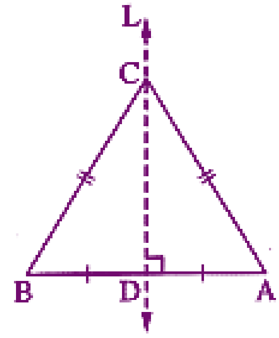


The converse of the previous property is true

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such that $CA = CB$, then the point C lies on the axis of $\overline{AB}$



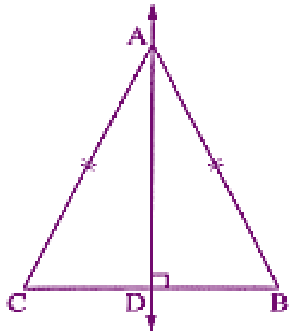
Axis of symmetry of Isosceles Triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC



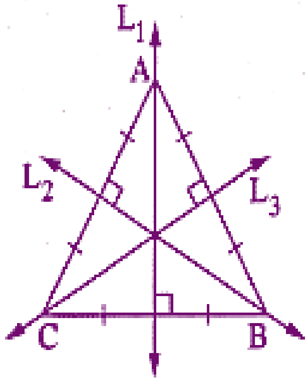
Axis of Symmetry of Equilateral Triangle

Remarks

1. The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2. The scalene triangle has no axes of symmetry.

The Figure	Number of axes of symmetry
Parallelogram – Trapezium – Scalene triangle	0
Isosceles triangle – Isosceles Trapezium	1
Rectangle – Rhombus	2
Equilateral triangle	3
Square	4
Regular pentagon	5
Regular hexagon	6
Circle	Infinite or very large

Exercises

EX. (1): Complete the following:

1	The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ....
2	The median of an isosceles triangle drawn from the vertex bisects .... and is perpendicular to ....
3	The bisector of the vertex angle of an isosceles triangle .... and ....
4	In Δ XYZ : If XY = XZ , $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of .... and ....
5	The straight line perpendicular to the midpoint of a line segment is called ....
6	In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ....
7	The number of axes of symmetry of the isosceles triangle equal ....
8	The number of symmetrical line in an scalene triangle = ...
9	The number of the axes of symmetry in an equilateral triangle = ...
10	The number of axes of symmetry of the triangle in which the measures of two angles are 50° , 70° = ...
11	In Δ ABC : If AB = AC , then the point A lies on the axis of symmetry of ....
12	If D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{CD} \perp \overline{AB}$, then CA = ...
13	The axis of symmetry of the line segment is the straight line which ....
14	Any point on the axis symmetry of a line segment is at two equal distance from ....
15	If the point A $\in$ the axis of symmetry of $\overline{BC}$, then AB = ...
16	The axis of symmetry of isosceles triangle is ....

EX. (2): Choose the correct answer:

1	The axis of symmetry of a line segment is the straight line which is ………. (a) Perpendicular to it. (b) its bisector. (c) parallel to it. (d) the perpendicular bisector.
2	If A $\in$ the axis of symmetry of $\overline{BC}$, then AB ………. AC (a) $\perp$ (b) $\equiv$ (c) // (d) =
3	If A lies on the axis of symmetry of $\overline{XY}$ then AX …………… AY (a) // (b) $\perp$ (c) = (d) $\neq$
4	The number of axis of symmetry in the scalene triangle is ………. (a) 1 (b) zero (c) 3 (d) 4
5	The number of axes of symmetry in the isosceles triangle is ………. (a) 1 (b) 2 (c) 3 (d) zero
6	The isosceles triangle has …………… axis (axes) of symmetry. (a) no (b) two (c) only one (d) three
7	The number of axes of symmetry in the equilateral triangle is ………. (a) 0 (b) 2 (c) 3 (d) 1
8	The equilateral triangle has ………. axes of symmetry. (a) one (b) two (c) three (d) otherwise
9	The triangle which has no axes of symmetry is ………. triangles. (a) scalene (b) isosceles (c) equilateral (d) otherwise
10	If Δ ABC has one axes of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ………. (a) 30° (b) 20° (c) 40° (d) 60°
11	The triangle which has three axes of symmetry is ………. triangle. (a) scalene (b) isosceles (c) right-angled (d) equilateral
12	Δ ABC in which m ($\angle$ A) = m ($\angle$ B) = 65° , then it has …………… axis (axes) of symmetry. (a) 1 (b) 2 (c) 3 (d) zero
13	In Δ ABC if : m ($\angle$ A) = 40° and m ($\angle$ B) = 70° , then Δ ABC has …………… axis (axes) of symmetry. (a) 3 (b) 1 (c) 2 (d) zero
14	The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may by ………. (a) a rhombus (b) a rectangle (c) a parallelogram (d) a trapezium

EX. (3): Answer the following:

1

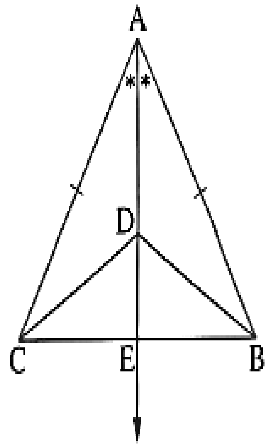
In the opposite figure :

ABC is a triangle in which AB = AC , $\overrightarrow{AE}$ bisects ∠ BAC ,

$\overline{AE} \cap \overline{BC} = \{E\}$ and $D \in \overline{AE}$

Prove that :

1 $BE = \frac{1}{2} BC$ 2 BD = CD



2

In the opposite figure :

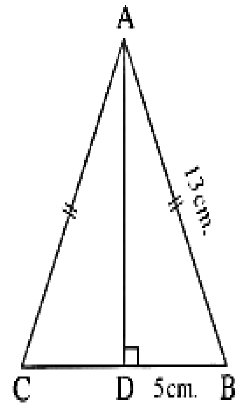
In Δ ABC , AB = AC , $\overline{AD} \perp \overline{BC}$, AB = 13 cm.

and BD = 5 cm.

Find :

1 The length of $\overline{BC}$

2 The area of Δ ABC



« 10 cm. , 60 cm^2 »

3

In the opposite figure :

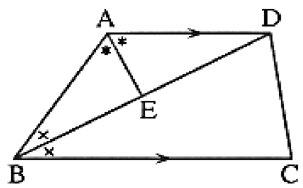
ABCD is a quadrilateral in which

$\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects ∠ ABC and

$\overrightarrow{AE}$ bisects ∠ BAD

Prove that :

1 AB = AD 2 $\overline{AE} \perp \overline{BD}$ 3 BE = ED



4

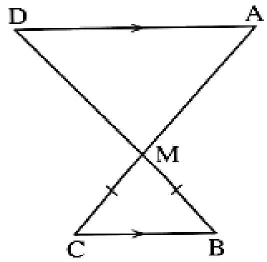
In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AD}$ // $\overline{BC}$ and MB = MC

Prove that :

1 Δ AMD is an isosceles triangle.

2 The axis of symmetry of Δ AMD is the same of Δ BMC



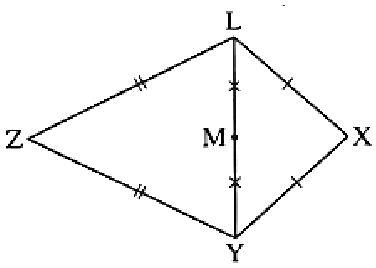
5

In the opposite figure :

XY = XL , ZY = ZL and LM = YM

Prove that :

X , M and Z are on the same straight line.



6

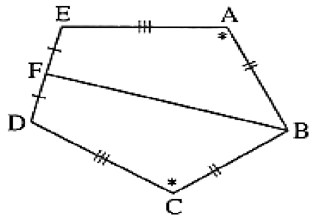
In the opposite figure :

AB = BC , AE = CD ,

m (∠ BAE) = m (∠ BCD)

and F is the midpoint of $\overline{DE}$

Prove that : $\overline{BF} \perp \overline{DE}$



7

In the opposite figure :

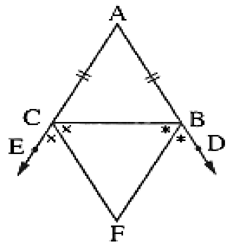
AB = AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects ∠ DBC and $\overrightarrow{CF}$ bisects ∠ BCE

Prove that :

1 Δ BFC is an isosceles triangle.

2 $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$



Explaining

Unit 5

Lesson (1)

Inequality

For any four numbers a , b , c and d :

1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a c > b c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

Remember that :

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

Lesson (2)

Comparing the measures of angles of triangle

Theorem

In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.

Remark

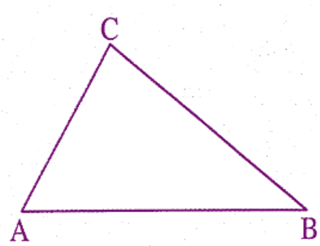
The greatest angle in measure of the triangle is opposite to the longest side of the triangle and its measure is greater than 60°.and the smallest angle in measure of the triangle is opposite to the shortest side of the triangle

and its measure is less than 60°

i.e. In ΔABC :

If $AB > BC > AC$, then $m(\angle C) > m(\angle A) > m(\angle B)$

, $m(\angle C) > 60°$ and $m(\angle B) < 60°$



EX. (1): Complete the following:

Exercises

1	The length of two sides in the triangle are not equal , then the greatest side in length is opposite to ……….
2	In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the ……….
3	In triangle ABC , if $BC > AB$, then $m(\angle A)$ ……… $m(\angle C)$
4	In ΔABC : $AB > AC$, then $m(\angle C)$ ………. $m(\angle B)$
5	In ΔABC , if $AB > BC > AC$, then the smallest angle in measure of it is angle ……….

EX. (2): Choose the correct answer:

1	In Δ ABC , AB > AC , then m (∠ C) ·········· m (∠ B) (a) < (b) > (c) = (d) ≤
2	In Δ ABC if AB > AC , then m (∠ B) ··············· m (∠ C) (a) > (b) < (c) = (d) ≥
3	In Δ ABC , AB > AC , m (∠ C) = 70° , then m (∠ B) may be ·········· (a) 70° (b) 50° (c) 80° (d) 75°
4	In Δ ABC : If BC > AB , then m (∠ A) ·········· m (∠ C) (a) = (b) < (c) ≤ (d) >
5	In the triangle XYZ , if XY > ZX , then m (∠ Y) ·········· m (∠ Z) (a) > (b) < (c) = (d) ≥
6	In Δ ABC : AB = AC , m (∠ B) = 65° , then : AC ·········· BC (a) < (b) > (c) = (d) ≤
7	In Δ ABC : If AB = 9 cm. , BC = 6 cm. , AC = 7 cm. , then the smallest angle is ·········· (a) ∠ BAC (b) ∠ ABC (c) ∠ ACB (d) ∠ BCA

EX. (3): Answer the following:

1

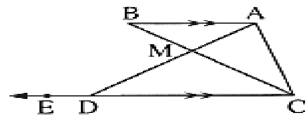
In the opposite figure :

$\overline{AB} \mathbin{//} \overline{CD}$, $\overline{AD} \cap \overline{BC} = \{M\}$, $E \in \overleftrightarrow{CD}$ and $E \notin \overline{CD}$

Prove that :

[1] m (∠ ACD) > m (∠ ABC)

[2] m (∠ ADE) > m (∠ ABC)



2

M is a point inside the triangle ABC

Prove that : m (∠ AMB) > m (∠ ACB)

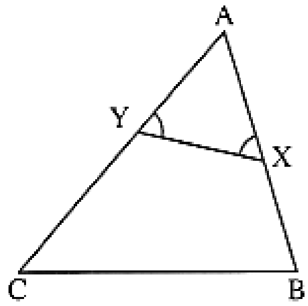
3

In the opposite figure :

ABC is a triangle in which : AC > AB , $X \in \overline{AB}$

and $Y \in \overline{AC}$ where m (∠ AXY) = m (∠ AYX)

Prove that : YC > XB



4

Arrange the measures of the angles of Δ ABC in each of the following cases ascendingly :

[1] If AB = 12 cm. , BC = 15 cm. and AC = 10 cm.

[2] If AB = 5.7 cm. , BC = 8.5 cm. and AC = 6 cm.

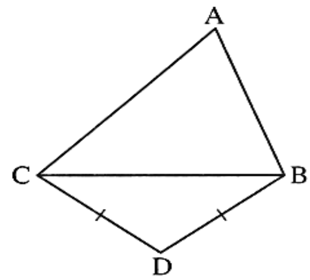
5

In the opposite figure :

AC > AB and DB = DC

Prove that :

m (∠ ABD) > m (∠ ACD)

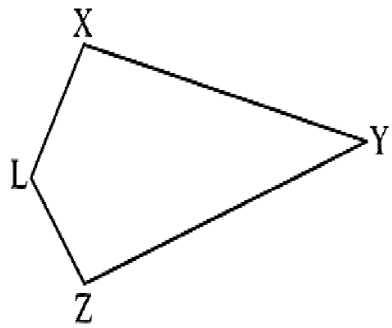


6

In the opposite figure :

XY > XL and YZ > ZL

Prove that : m (∠ XLZ) > m (∠ XYZ)



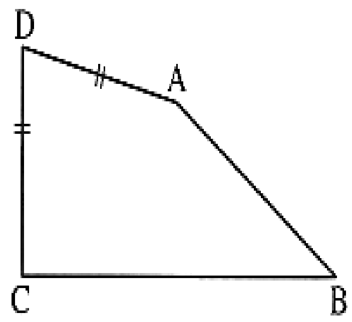
7

In the opposite figure :

ABCD is a quadrilateral in which :

AD = DC and BC > AB

Prove that : m (∠ A) > m (∠ C)



8

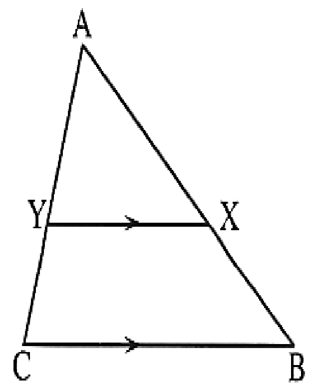
In the opposite figure :

ABC is a triangle ,

AB > AC and $\overline{XY}$ // $\overline{BC}$

Prove that :

m (∠ AYX) > m (∠ AXY)



9

In the opposite figure :

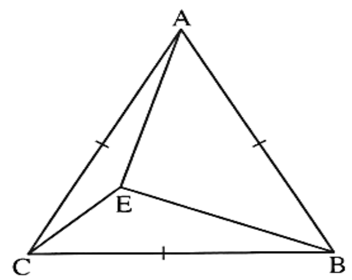
ABC is an equilateral triangle ,

E is a point inside it ,

m (∠ ECB) > m (∠ EBC)

Prove that : 1 m (∠ ABE) > m (∠ ACE)

2 m (∠ A) > m (∠ ABE) > m (∠ ACE)

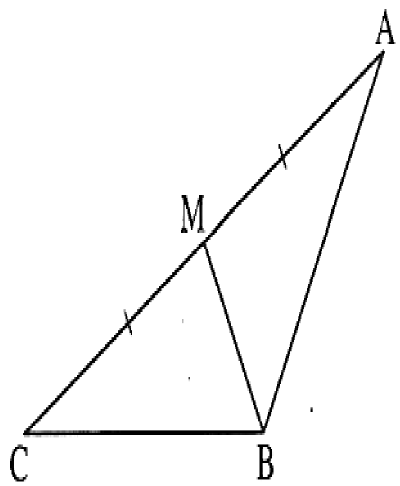


10

In the opposite figure :

$\overline{BM}$ is a median in the triangle ABC and BM < AM

Prove that : ∠ ABC is an obtuse angle.



Explaining

Lesson (3)

Comparing the lengths of sides in a triangle

Theorem

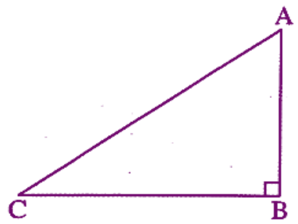
In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Corollaries

Corollary 1

In the right-angled triangle , the hypotenuse is the longest side.

In the opposite figure :



If ΔABC is right-angled at B , then $m(\angle B) > m(\angle A)$,
$m(\angle B) > m(\angle C)$ because $\angle B$ is a right angle and each of $\angle A$ and $\angle C$ is acute , so we find that :
$AC > BC$ and $AC > AB$ (according to the previous theorem).

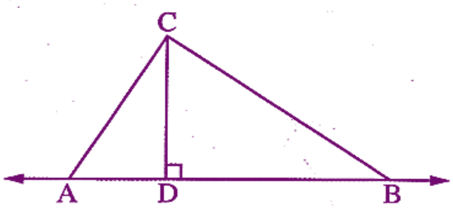
Notice that :

In the obtuse-angled triangle, the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

In the opposite figure :



If $C \notin \overleftrightarrow{AB}$ and $D \in \overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$,
then $\overline{CB}$ is the hypotenuse in ΔCBD
which is right-angled at D ,
$\overline{CA}$ is the hypotenuse in ΔCDA which is right-angled at D and so on ...
According to corollary 1 , we find that $CB > CD$, $CA > CD$ and so on ...
i.e. $CD < CB$ and $CD < CA$

Definition

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

EX. (1): Complete the following:

Exercises

1	In the right-angled triangle the longest side in it called ⋯⋯⋯
2	The longest side length in the right-angled triangle is ⋯⋯⋯
3	If XYZ is a right-angled triangle at Y , then the longest side is ⋯⋯⋯
4	In a triangle if two angles have unequal in measure , then the greatest angle in measure is ⋯⋯⋯
5	The smallest angle of a triangle (in measure) is opposite to ⋯⋯⋯
6	In any triangle the greatest angle in measure is opposite to ⋯⋯⋯
7	If ABC is an obtuse-angled triangle at C , then AB ⋯⋯⋯ BC
8	If : $x > y$, z is positive number then : $x z >$ ⋯⋯⋯
9	Δ ABC in which : m ($\angle$ A) = 100° , then the greatest side in length is ⋯⋯⋯
10	The longest side in the triangle ABC in which m ($\angle$ B) = 105° is ⋯⋯⋯
11	Δ ABC in which m ($\angle$ A) = 110° , then the greatest side in length is ⋯⋯⋯
12	Δ ABC in which : m ($\angle$ C) = 112° , then the longest side is ⋯⋯⋯
13	In Δ ABC : If m ($\angle$ B) = 120° , then the longest side in Δ ABC is ⋯⋯⋯
14	In Δ DEF if m ($\angle$ E) = 125° , then the longest side in this triangle is ⋯⋯⋯
15	In Δ ABC : If m ($\angle$ A) = 130° , then the longest side is ⋯⋯⋯
16	In triangle ABC , if m ($\angle$ A) = 70° , m ($\angle$ B) = 30° , then the longest side in length is ⋯⋯⋯
17	In Δ ABC : if m ($\angle$ A) = 67° and m ($\angle$ B) = 33° , then AB > ⋯⋯⋯ > ⋯⋯⋯

18	Δ ABC in which m ($\angle$ B) = 70° and m ($\angle$ C) = 35° the longest side in length is ·······
19	In Δ ABC , m ($\angle$ A) = 50° , m ($\angle$ B) = 65° , then the number of axes of symmetry equals ··········
20	In Δ ABC if : m ($\angle$ A) = 50° and m ($\angle$ B) = 60° , then the longest side in this triangle is ···············
21	In Δ ABC m ($\angle$ B) = 70° and m ($\angle$ C) = 60° then AC ··············· AB
22	In the isosceles triangle if : AB = AC , m ($\angle$ A) = 70° , then AB < ···············
23	In the triangle ABC : if m ($\angle$ B) – m ($\angle$ A) > m ($\angle$ C) , then AC ·········· AB

EX. (2): Choose the correct answer:

1	ΔXYZ, $m(\angle X) = 60°$, $m(\angle Y) = 40°$, then XZ .... XY (a) < (b) > (c) = (d) nothing.
2	ABC is a triangle in which : $m(\angle B) = 70°$, $m(\angle C) = 50°$, then AC .... AB (a) > (b) < (c) = (d) ≡
3	In a triangle ABC : $m(\angle B) = 75°$, $m(\angle C) = 50°$, then BC .... AB (a) < (b) > (c) = (d) ≡
4	ABC is a triangle in which : $m(\angle B) = 80°$, $m(\angle C) = 50°$, then BC .... AB (a) > (b) < (c) = (d) ≡
5	If : $m(\angle A) = 50°$ and $m(\angle B) = 60°$ in triangle ABC then AB ....AC (a) > (b) < (c) = (d) ≤
6	In a triangle ABC : If $m(\angle A) = 80°$, $m(\angle C) = 60°$, then AB .... BC (a) > (b) < (c) = (d) ≥
7	Triangle ABC : If $m(\angle B) = 70°$, $m(\angle C) = 60°$, then BC .... AB (a) < (b) > (c) = (d) ≥
8	In ΔLMN, if $m(\angle N) = 75°$, $m(\angle M) = 60°$, then LM .... LN. (a) > (b) < (c) = (d) twice
9	ΔABC, $m(\angle B) = 90°$, then AB .... AC (a) > (b) = (c) < (d) ≥
10	In ΔXYZ : If $m(\angle X) = 30°$ and $m(\angle Y) = 80°$, then .... (a) XY < XZ (b) XY > XZ (c) XY = XZ (d) XY < YZ
11	In ΔABC : $m(\angle A) = 60°$ and $m(\angle C) = 45°$, then .... (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$

12	KLM is a triangle in which m ($\angle$ K) = 50° and m ($\angle$ M) = 60° , then which of the following statement is true ? …………… (a) KL = KM (b) KM > KL (c) KM < ML (d) LM > KL
13	The triangle in which the measure of two angles are 74° and 53° is …………… triangle. (a) a right-angled (b) an isosceles (c) an equilateral (d) a scalene
14	In Δ ABC : If AB > AC , m ($\angle$ C) = 70° , then m ($\angle$ B) may equal ………. (a) 70° (b) 50° (c) 80° (d) 75°
15	In Δ ABC , if m ($\angle$ A) = 50° and m ($\angle$ B) = 30° , then the shortest side in the triangle ABC is …………… (a) $\overline{AB}$ (b) $\overline{CB}$ (c) $\overline{AC}$ (d) $\overleftrightarrow{BC}$
16	Δ ABC which : m ($\angle$ A) = 50° , m ($\angle$ B) = 60° the longest side of it is ………. (a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) $\overline{CB}$
17	In Δ ABC if : m ($\angle$ B) = 60° and m ($\angle$ C) = 50° , then the shortest side in triangle ABC is …………… (a) $\overline{AC}$ (b) $\overleftrightarrow{BC}$ (c) $\overline{BC}$ (d) $\overline{AB}$
18	In the triangle ABC , if m ($\angle$ B) = 90° , then the greatest side in length is …………… (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{XY}$
19	In Δ ABC if : m ($\angle$ B) = 130° , then the longest side of it is …………… (a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) it's median
20	In the triangle ABC : If m ($\angle$ B) > m ($\angle$ C) , then AB ………. AC (a) < (b) > (c) = (d) otherwise
21	In Δ ABC : if m ($\angle$ B) > m ($\angle$ C) , then AC ………. AB (a) > (b) < (c) = (d) ≤
22	In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then …………… (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{AC}$
23	In Δ ABC : m ($\angle$ A) < m ($\angle$ C) < m ($\angle$ B) , then ………. (a) AB > AC (b) BC > AC (c) AC > AB (d) BC > AB
24	The triangle ABC is obtuse-angled triangle at B , then the longest side is ………. (a) AB (b) BC (c) AC (d) AD
25	Δ XYZ is right-angled at Y , then XZ ………. YZ (a) = (b) > (c) ≤ (d) <
26	In Δ ABC : m ($\angle$ B) + m ($\angle$ C) = 3 m ($\angle$ A) , then m ($\angle$ A) = ……………° (a) 30 (b) 60 (c) 45 (d) 90

EX. (3): Answer the following:

1

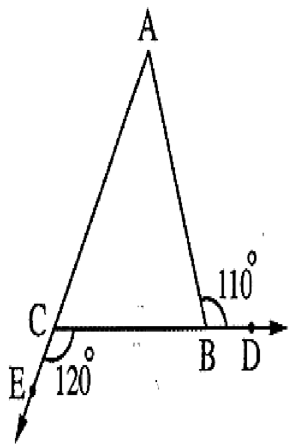
In the opposite figure :

ABC is a triangle , $D \in \overrightarrow{CB}$,

$E \in \overrightarrow{AC}$, $m(\angle ABD) = 110°$

and $m(\angle BCE) = 120°$

Prove that : $AB > BC$



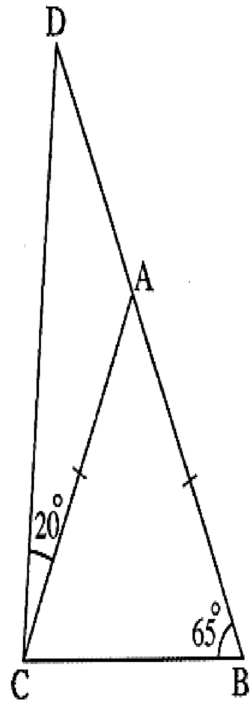
2

In the opposite figure :

$AB = AC$, $m(\angle ABC) = 65°$

, $m(\angle ACD) = 20°$, $A \in \overline{BD}$

Prove that : $AB > AD$



3

ABC is a triangle , $\overrightarrow{CD}$ bisects $\angle C$, $\overrightarrow{CD} \cap \overline{AB} = \{D\}$

Prove that : $BC > BD$

4

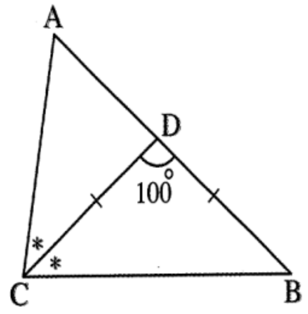
In the opposite figure :

ABC is a triangle , $\overrightarrow{CD}$ bisects $\angle$ C and intersects $\overline{AB}$ at point D

, m ($\angle$ BDC) = 100° and DB = DC

Prove that :

AC > DB



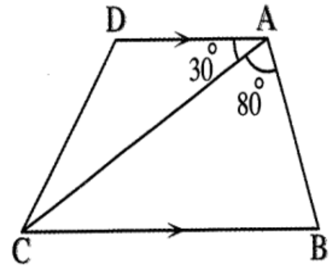
5

In the opposite figure :

$\overrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 80° and m ($\angle$ DAC) = 30°

Prove that :

BC > AB



6

In the opposite figure :

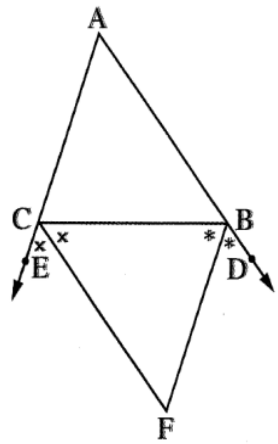
ABC is a triangle in which : AB > AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$

, $\overrightarrow{BF}$ bisects $\angle$ DBC and $\overrightarrow{CF}$ bisects $\angle$ BCE

, $\overrightarrow{BF} \cap \overrightarrow{CF}$ = {F}

Prove that :

(1) m ($\angle$ FBC) > m ($\angle$ BCF) (2) CF > BF

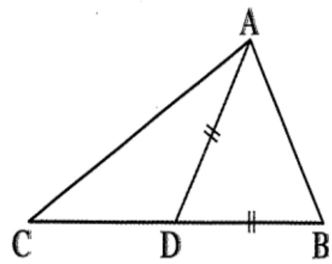


7

In the opposite figure :

ABC is a triangle and D $\in \overline{BC}$ where BD = AD

Prove that : BC > AC

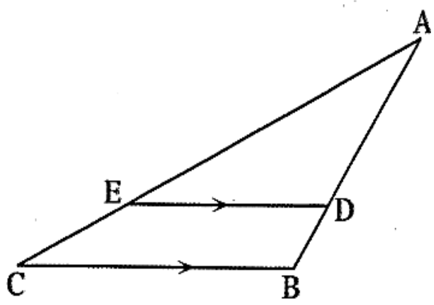
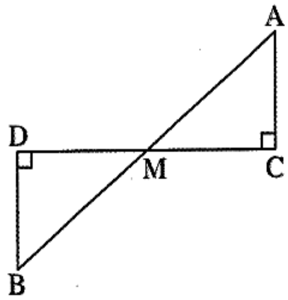


8

ABC is a triangle in which : m ($\angle$ A) = $(5X + 2)°$,

m ($\angle$ B) = $(6X - 10)°$ and m ($\angle$ C) = $(X + 20)°$

Order the lengths of sides of the triangle ascendingly.

9	ABC is a triangle in which : m (∠ A) = 40° and m (∠ B) = 75° Order the lengths of the sides of the triangle descendingly.
10	**In the opposite figure :** ABC is an obtuse-angled triangle at B , $\overline{DE} \parallel \overline{BC}$ **Prove that :** AE > AD 
11	ABC is a right-angled triangle at B , D ∈ $\overline{AC}$ and E ∈ $\overline{BC}$ where AD = BE **Prove that :** m (∠ CED) > m (∠ CDE)
12	**In the opposite figure :** $\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$ **Prove that :** AB > CD 

LESSON (4) Triangle inequality

Generally

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

Generally

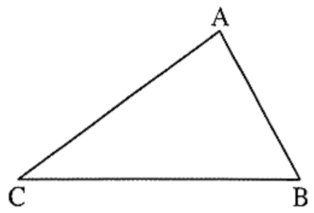
In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

i.e. In any triangle such as Δ ABC

, we get : $AB + BC > AC$

, $BC + CA > AB$

, $CA + AB > CB$



Corollary

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

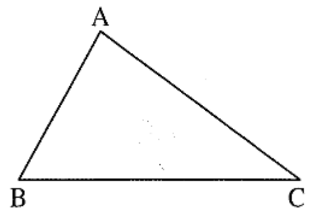
And you can prove that from the triangle inequality as follows :

In any triangle ABC :

$AC + AB > BC$ (1)

, $\because AB + BC > AC$ **i.e.** $BC > AC - AB$ (2)

From (1) and (2) , we deduce that : $\boxed{AC - AB < BC < AC + AB}$



Remark

To check the possibility that three lengths can be side lengths of a triangle , do as follows :

Compare the greatest length with the sum of the other two lengths :

- If the greatest length is greater than or equal to the sum of the other two lengths , you deduce that the three given lengths couldn't be lengths of the three sides of a triangle. (**i.e.** no triangle could be drawn with these side lengths).
- If the greatest length is less than the sum of the other two lengths , you deduce that the three given lengths could be lengths of the three sides of a triangle. (**i.e.** a triangle could be drawn with these side lengths).

Exercises

EX. (1): Complete the following:

1	In any triangle the sum of any two sides ……… the length of the third side.
2	In Δ ABC , AB + BC > ………
3	The length of any side in a triangle ……… the sum of the lengths of the two other sides.
4	ABC is a triangle , if AB = 3 cm. and BC = 5 cm. , then AC $\in$] ……… , ……… [
5	Δ XYZ in which XY = 4 cm. and YZ = 3 cm. , then XZ $\in$] …………… , ……………[
6	If x cm. , 4 cm. and 5 cm. are lengths of the sides of a triangle , then ……… < x < ………
7	If : x , 8 , 7 cm. are lengths of the sides of a triangle then …………… < x < ……………
8	If the lengths of two sides in triangle are 3 cm. and 9 cm. , then …………… < the length of third side < ……………
9	If the length of two sides of an isosceles triangle are 4 cm. and 10 cm. , then the length of the third side is ………
10	If the length of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side = ……… cm.
11	The length of two sides in an isosceles triangle are 8 cm. , 4 cm. then the length of the third side = ……………
12	If the lengths of two sides in an isosceles triangle are 6 cm. , 3 cm. , then the lengths of the third side is …………… cm.
13	The length of two sides in the isosceles triangle are 3 cm. and 8 cm. , then the length of third side equals ……… cm.
14	The triangle whose side lengths are (2 x – 1) cm. , (x + 3) cm. , and 7 cm. becomes an equilateral triangle when x = …………… cm.

EX. (2): Choose the correct answer:

1	The sum of lengths of any two sides in any triangle ……… the length of the third side. (a) is less than (b) is greater than (c) equals (d) otherwise
2	If the lengths of two sides in an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is …………… cm. (a) 2 (b) 3 (c) 5 (d) 7
3	Δ ABC , AB = 2 cm. , BC = 7 cm. , then AC may equal ……… (a) 2 cm. (b) 5 cm. (c) 9 cm. (d) 8 cm.
4	The numbers 6 , 3 , ……… can be lengths of sides of an isosceles triangle. (a) 3 (b) 6 (c) 9 (d) 11
5	If the lengths of two sides in the isosceles triangle are 3 cm. , 7 cm. , then the length of third side = ……… (a) 3 cm. (b) 7 cm. (c) 10 cm. (d) 4 cm.
6	The numbers 7 , 3 and ……… can be lengths of sides of an isosceles triangle. (a) 3 (b) 10 (c) 7 (d) 4
7	If 3 cm. , 7 cm. are two side lengths in a triangle , then the smallest number of third side is ……… (a) 3 cm. (b) 4 cm. (c) 5 cm. (d) 6 cm.
8	The numbers 5 , 4 and ……… can be lengths of sides of a triangle. (a) 8 (b) 9 (c) 10 (d) 12
9	The numbers 4 , 8 , …………… can be lengths of sides of an isosceles triangle. (a) 4 (b) 8 (c) 12 (d) 3
10	If any sides in isosceles triangle 8 cm. , 4 cm. , then the length of the third side is ……… (a) 8 (b) 4 (c) 12 (d) 5

11 The lengths of two sides in a triangle are 4 cm. and 9 cm. and it has on axis of symmetry , then the length of third side is ....

(a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

12 The lengths of 5 cm. , 6 cm. and .... can be length of the sides of a triangle.

(a) 15 cm. (b) 13 cm. (c) 11 cm. (d) 8 cm.

13 The numbers 5 , 7 , ... can be lengths of sides of triangle.

(a) 12 (b) 3 (c) 2 (d) 13

14 3 , 10 , ... can be lengths of sides of an isosceles triangle.

(a) 10 (b) 8 (c) 6 (d) 4

15 In Δ ABC if : AB = 3 cm. and BC = 5 cm. , then AC $\in$ ....

(a) $]3 , 8]$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $]2 , 5[$

16 In the triangle ABC , if BC = 9 cm. , AB = 7 cm. , then m ($\angle$ C) .... m ($\angle$ A)

(a) = (b) ≥ (c) > (d) <

17 Which of the following can be sides to draw the triangle ....

(a) 5 cm. , 6 cm. , 12 cm. (b) 5 cm. , 6 cm. , 11 cm.

(c) 5 cm. , 6 cm. , 4 cm. (d) 4 cm. , 6 cm. , 10 cm.

18 Which of the following numbers can be the lengths of sides of a triangle ?

(a) 4 , 6 , 10 (b) 4 , 6 , 8 (c) 2 , 3 , 6 (d) 4 , 5 , 10

19 The lengths which can be the lengths of the sides of a triangle are ....

(a) 3 , 4 , 7 (b) 3 , 3 , 6 (c) 3 , 5 , 7 (d) 1 , 5 , 7

20 Which of the following set of numbers can be lengths of sides of a triangle ....

(a) 2 , 3 , 6 (b) 2 , 3 , 5 (c) 2 , 3 , 4 (d) 2 , 3 , 7

21 How many different triangles can be formed with sides of lengths a whole number of cm. and each with perimeter 7 cm. ?

(a) 1 (b) 2 (c) 3 (d) 4

22 If the length of one side of a triangle is 5 cm. , then which of the following could be the lengths of the other two sides ?

(a) 2 cm. and 3 cm. (b) 7 cm. and 2 cm.
(c) 2 cm. and 2 cm. (d) 4 cm. and 6 cm.

23 Which of the following numbers cannot be the lengths of sides of a triangle ....

(a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5

24 In any triangle ABC : AB .... BC – AC

(a) > (b) < (c) = (d) ≤

25 In the triangle ABC , AC .... (AB – BC)

(a) > (b) ≥ (c) ≤ (d) <

26 In any triangle ABC , AB + BC .... AC

(a) = (b) < (c) > (d) ≤

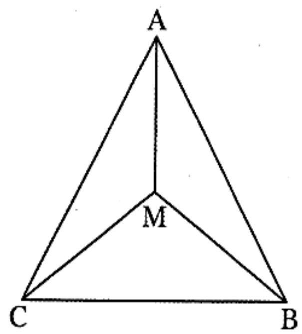
EX. (3): Answer the following:

1 In the opposite figure :

ABC is a triangle in which M is a point inside it.

Prove that :

$MA + MB + MC > \frac{1}{2}$ the perimeter of the triangle ABC



2 Prove that the length of any side in a triangle is less than half of the perimeter.

3 Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.

Guide Answers

Algebra

Unit 1

Lesson 1

Ex.(1):complete

1	a	8	64	15	-8	22	{1,-1,-5}
2	-2	9	64	16	12	23	{5}
3	5	10	16	17	Ø	24	{1}
4	25	11	32	18	Ø	25	{0,1}
5	6	12	0 or 20	19	Ø	26	{-1}
6	0	13	16	20	Ø		
7	61	14	512	21	Ø		

Ex.(2):choose

1	C	8	C	15	A
2	C	9	C	16	B
3	C	10	C	17	C
4	A	11	B	18	D
5	B	12	B	19	B
6	B	13	C	20	C
7	A	14	D		

Ex.(3):Answer the following

1	$X = 5^3 = 125$
2	$X = \sqrt[3]{-8} = -2$
3	$\because x^3 = -27$ $\therefore x = \sqrt[3]{-27} = -3$ $\therefore$ The S.S. $= \{-3\}$
4	$\because 8x^3 = 8 - 7 = 1$ $\therefore x^3 = \frac{1}{8}$ $\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\therefore$ The S.S. $= \{\frac{1}{2}\}$
5	$\because x + 3 = \sqrt[3]{343} = 7$ $\therefore x = 7 - 3 = 4$ $\therefore$ The S.S. $= \{4\}$
6	$\because (5x-2)^3 = 18 - 10 = 8$ $\therefore 5x - 2 = \sqrt[3]{8} = 2$ $\therefore 5x = 2 + 2 = 4$ $\therefore x = \frac{4}{5}$ $\therefore$ The S.S. $= \{\frac{4}{5}\}$
7	The edge length of the cube $= \sqrt[3]{15\frac{5}{8}} = \sqrt[3]{\frac{125}{8}} = 2.5$ cm.
8	The length of the inner edge $= \sqrt[3]{1000} = 10$ cm.
9	The volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{1372}{81}\pi$ $\therefore r = \sqrt[3]{\frac{343}{27}} = \frac{7}{3}$ $\therefore r^3 = \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27}$ $\therefore$ The diameter length of the sphere $= 2 \times \frac{7}{3} = \frac{14}{3}$ length unit.

Lessons 2,3

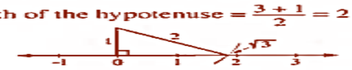
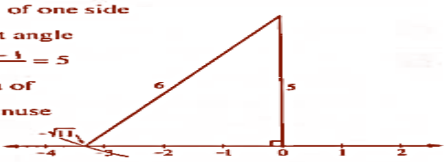
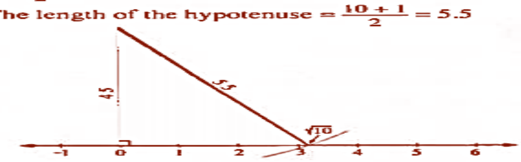
Ex.(1):complete

1	-3
2	3
3	4
4	4
5	3
6	Ø , Q
7	R , Q`
8	R-{0} , Ø
9	$2\sqrt{5}$
10	$\sqrt{3} - \sqrt{2}$

Ex.(2):choose

1	C	8	C	15	B
2	B	9	A	16	C
3	C	10	D	17	B
4	D	11	B	18	C
5	B	12	B	19	C
6	D	13	C		
7	C	14	B		

Ex.(3):Answer the following

1	3.32
2	-2.1
3	$\because \sqrt{1} < \sqrt{2} < \sqrt{4}$ $\therefore 1 < \sqrt{2} < 2$ $\therefore x = 1$
4	$\because \sqrt{64} < \sqrt{80} < \sqrt{81}$ $\therefore 8 < \sqrt{80} < 9$ $\therefore x = 8$
5	$\because (1.4)^2 = 1.96$, $(1.5)^2 = 2.25$, $(\sqrt{2})^2 = 2$ $\therefore \sqrt{2}$ is included between 1.4 , 1.5
6	$\because (2.4)^3 = 13.824$, $(2.5)^3 = 15.625$ $(\sqrt[3]{15})^3 = 15$ $\therefore \sqrt[3]{15}$ is included between 2.4 , 2.5
7	$\boxed{1}$ The length of one side of the right angle $= \frac{3-1}{2} = 1$ The length of the hypotenuse $= \frac{3+1}{2} = 2$  $\boxed{2}$ The length of one side of the right angle equals $\frac{11-1}{2} = 5$ The length of the hypotenuse $= \frac{11+1}{2} = 6$  $\boxed{3}$ The length of one side of the right angle $= \frac{10-1}{2} = 4.5$ The length of the hypotenuse $= \frac{10+1}{2} = 5.5$ 
8	The side length $= \sqrt{5}$ cm. , $\sqrt{5} \notin \mathbb{Q}$
9	The edge length $= \sqrt[3]{1.728} = \frac{6}{5}$ cm. , $\frac{6}{5} \in \mathbb{Q}$
10	$\boxed{1}$ The ascending order is : $-\sqrt{11}$, $-\sqrt{7}$, $-\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$ and $\sqrt{15}$ $\boxed{2}$ $\because 0.6 = \sqrt{0.36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$ $\therefore$ The ascending order is : $-\sqrt{45}$, $-\sqrt{1}$, $\sqrt{0.36}$, $\sqrt{20}$ and $\sqrt{27}$ i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0.6 , $\sqrt{20}$ and $\sqrt{27}$
11	$\boxed{1}$ > $\boxed{2}$ > $\boxed{3}$ < $\boxed{4}$ < $\boxed{5}$ > $\boxed{6}$ =
12	$\because$ The total area of the cube $= 6\ell^2$ $\therefore 13.5 = 6\ell^2$ $\therefore \frac{13.5}{6} = \ell^2$ $\therefore \ell = \sqrt{\frac{13.5}{6}} = 1.5$ cm. , $1.5 \in \mathbb{Q}$
13	$\boxed{1}$ $\because 8 = \sqrt{64}$ $\therefore$ The descending order is : $\sqrt{70}$, $\sqrt{64}$, $\sqrt{62}$ and $-\sqrt{50}$ i.e. $\sqrt{70}$, 8 , $\sqrt{62}$ and $-\sqrt{50}$ $\boxed{2}$ $\because 9 = \sqrt{81}$ $\therefore$ The descending order is : $\sqrt{101}$, $\sqrt{81}$, $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$ i.e. $\sqrt{101}$, 9 , $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$
14	$\boxed{1}$ $x^2 = \frac{10}{5} = 2$ $\therefore x = \pm\sqrt{2}$ $\therefore x \in \mathbb{Q}'$ $\boxed{2}$ $x^2 = \frac{9}{4}$ $\therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\therefore x \in \mathbb{Q}$ $\boxed{3}$ $x = \sqrt[3]{125}$ $\therefore x = 5$ $\therefore x \in \mathbb{Q}$ $\boxed{4}$ $x^3 = \frac{27}{3} = 9$ $\therefore x = \sqrt[3]{9}$ $\therefore x \in \mathbb{Q}'$

Lesson 4

Ex.(1):complete

1	[1,5[
2	]1,5]
3	]-∞,∞[
4	[3,5]
5	[-2,2]
6	[5,7]
7	Ø
8	]3,5[
9	[-1,1[
10	]4,5]
11	]0,3[
12	]1,5[
13	[2,5[
14	]2,5[
15	]3,4[
16	]3,5]
17	]5,7[
18	{-4,6}
19	{-1,5}
20	{2,7}

Ex.(3):Answer the following

1	Use the number line to get the following results: (1) [-1,∞[(2) [3,4] (3) [-1,4[- {3}
2	(1) [0,1] (2) [-2,∞[
3	(1)]-4,∞[(2) [3,8[(3)]-∞,3[
4	(1) [2,4] (2) [-1,7]
5	(1) [0,1] (2) [-2,∞[(3)]1,∞[
6	Use the number line to get the following results: (1) [-1,∞[(2) [-1,3[
7	(1) [0,8[(2) [1,3]
8	Use the number line to get the following results: [-2,5] , [-2,1[
9	Use the number line to get the following results: (1)]2,4] (2) [-2,2]
10	Use the number line to get the following results: (1) [-1,7] (2) [2,4] (3) [-1,2[
11	Use the number line to get the following results: (1) [2,5] (2) [1,7]

Ex.(2):choose

1	A	8	D
2	D	9	A
3	C	10	C
4	C	11	B
5	A	12	C
6	B	13	C
7	C	14	C

Lessons 5,6

Ex.(1):complete

1	1 , 0	8	$\frac{\sqrt{2}}{2}$	15	$\sqrt{2}$
2	$\sqrt{2}$-1	9	1	16	$\sqrt{3}$
3	$\sqrt{2}-\sqrt{5}$	10	1	17	-2
4	$\sqrt{3}-\sqrt{7}$	11	additive inverse	18	$\sqrt{125}$
5	$5\sqrt{3}-\sqrt{48}$	12	R	19	$\pm\frac{2\sqrt{2}}{3}$
6	$5\sqrt{3}$	13	400	20	20 , 0
7	$\frac{\sqrt{3}}{3}$	14	$\frac{1}{2}$	21	$3+\sqrt{2}$

Ex.(2):choose

1	D	10	B
2	C	11	C
3	A	12	B
4	D	13	A
5	D	14	B
6	B	15	B
7	C	16	B
8	D	17	C
9	b	18	A

Ex.(3):Answer the following

1	(1) $2\sqrt{2}+2\sqrt{5}$ (2) $5\sqrt{2}+2$ (3) $7+2\sqrt{7}$ (4) $5\sqrt{3}+3$ (5) 2-1=1 (6) $\sqrt{5}-7$
2	$\sqrt{25\times 3}-2\sqrt{9\times 3}+\sqrt{9\times\frac{1}{3}}=5\sqrt{3}-6\sqrt{3}+\sqrt{3}=0$
3	$2\times 3\sqrt{2}+5\sqrt{2}+\frac{1}{3}\times 9\sqrt{2}$ $=6\sqrt{2}+5\sqrt{2}+3\sqrt{2}=14\sqrt{2}$
4	$\sqrt{25\times 2}+\sqrt{9\times 2}-\sqrt{2}=5\sqrt{2}+3\sqrt{2}-\sqrt{2}=7\sqrt{2}$
5	$3\sqrt{3}+5\times 3\sqrt{2}-10\sqrt{3}=15\sqrt{2}-7\sqrt{3}$
6	$2\sqrt{5}+2\sqrt{3}-2\sqrt{3}-\sqrt{5}=\sqrt{5}$
7	$\boxed{1}$ $5\sqrt{2}+2\sqrt{2}=7\sqrt{2}$ $\boxed{2}$ $2\sqrt{5}-3\sqrt{5}=-\sqrt{5}$
8	$\boxed{1}$ $x+y=3+\sqrt{5}+1-\sqrt{5}=4$ $x\times y=(3+\sqrt{5})(1-\sqrt{5})=3-2\sqrt{5}-5$ $=-2-2\sqrt{5}$ $\boxed{2}$ $x+y=\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$ $x\times y=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$ $\boxed{3}$ $x+y=5-3\sqrt{2}+5-3\sqrt{2}=10-6\sqrt{2}$ $x\times y=(5-3\sqrt{2})(5-3\sqrt{2})$ $=25-30\sqrt{2}+18=43-30\sqrt{2}$

Lesson 7

Ex.(1):complete

1	-1	8	$1-\sqrt{7}$	15	$\sqrt{5}+2$
2	-3	9	$\sqrt{3}+5$	16	$(-1, 2\sqrt{3})$
3	1	10	$\sqrt{5}-\sqrt{3}$	17	24
4	5	11	$3-\sqrt{2}, 7$	18	$6\sqrt{5}$
5	4	12	$\sqrt{3}-\sqrt{2}$	19	$\frac{2\sqrt{2}-\sqrt{5}}{3}$
6	6	13	2	20	4
7	$\sqrt{3}+\sqrt{2}$	14	20	21	20

Ex.(2):choose

1	B	5	B
2	C	6	B
3	C	7	B
4	A	8	B

Ex.(3):Answer the following

1

$(1) 16-18=-2$ $\qquad$ $(2) 3+\sqrt{3}-2=\sqrt{3}+1$

2

(1) $\because X=\sqrt{5}+\sqrt{2}$, $y=\frac{3}{\sqrt{5}+\sqrt{2}}$ $\therefore y=\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{3(\sqrt{5}-\sqrt{2})}{3}=\sqrt{5}-\sqrt{2}$

$\therefore$ x and y are conjugate numbers

(2) $\frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\frac{2\sqrt{5}}{3}$

3

$b=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\sqrt{2}-1$, $(a-b)^2=(\sqrt{2}+1-(\sqrt{2}-1))^2=(2)^2=4$

4

$\because x=3+\sqrt{5}$, $y=\frac{4}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{4(3-\sqrt{5})}{9-5}$

$=3-\sqrt{5}$

$\therefore$ x and y are two conjugate numbers

$\boxed{1}$ $xy=(3+\sqrt{5})(3-\sqrt{5})=9-5=4$

$\boxed{2}$ $x^2+y^2=(x+y)^2-2xy$

$=(3+\sqrt{5}+3-\sqrt{5})^2-2\times4=36-8=28$

5

$\because b=\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{3-2}$

$=\sqrt{3}-\sqrt{2}$

We know that : $(a-b)(a+b)=a^2-b^2$

$\therefore(\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$=2\sqrt{2}\times2\sqrt{3}=4\sqrt{6}$

6

$\frac{x+y}{xy-1}=\frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

$=\frac{2\sqrt{5}}{5-2-1}=\frac{2\sqrt{5}}{2}=\sqrt{5}$

7	$\because y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$ $= \sqrt{5}+\sqrt{3}$ $\therefore x^2 + 2xy + y^2 = (x+y)^2$ $= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20$
8	$x^2 y^2 = (xy)^2 = \left(\frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{4}{\sqrt{7}+\sqrt{3}}\right)^2$ $= \left(\frac{16}{7-3}\right)^2 = 4^2 = 16$
9	$\because x = \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}+15}{5}$ $= \sqrt{10}+3$ $, y = \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}-6}{2} = \sqrt{10}-3$ $\boxed{1}$ $x^2 + y^2 = (x+y)^2 - 2xy$ $= (\sqrt{10}+3+\sqrt{10}-3)^2 - 2(\sqrt{10}+3)(\sqrt{10}-3)$ $= (2\sqrt{10})^2 - 2 \times (10-9) = 40 - 2 = 38$ $\boxed{2}$ $xy = (\sqrt{10}+3)(\sqrt{10}-3) = 10 - 9 = 1$ $\therefore x^2 + y^2 = 38\,xy$
10	$\because a = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$ $= \sqrt{7}+\sqrt{3}$ $, b = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3}$ $= \sqrt{7}-\sqrt{3}$ $\therefore \frac{a-b}{ab} = \frac{\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{3}}{7-3} = \frac{\sqrt{3}}{2}$
11	$\because x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$ $, y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ $\therefore x^2 + y = (2-\sqrt{3})^2 + 4\sqrt{3}$ $= 4 - 4\sqrt{3} + 3 + 4\sqrt{3} = 7$
12	$\because y = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$ $= \sqrt{5}+\sqrt{2}$ $, x = \sqrt{5}-\sqrt{2}$ $\therefore$ x and y are two conjugate numbers. $\therefore x^2 - 2xy + y^2 = (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2$ $= (-2\sqrt{2})^2 = 8$

Lesson 8

Ex.(1):complete

1	16	8	$\frac{\sqrt{2}}{2}$
2	24	9	additive inverse
3	24	10	R
4	-2	11	400
5	$\sqrt[3]{50}$	12	9
6	$1 , 0$	13	$\frac{-1}{4}$
7	$\sqrt{2} - 1$	14	$\frac{2\sqrt[3]{7}}{3}$

Ex.(2):choose

1	C	6	C
2	A	7	A
3	A	8	D
4	C	9	B
5	B		

Ex.(3):Answer the following

1	$3\sqrt[3]{2} - 2\sqrt[3]{2} + \sqrt[3]{2} = 2\sqrt[3]{2}$
2	$3\sqrt[3]{2} + \sqrt[3]{2} - 4\sqrt[3]{2} = 0$
3	$6\sqrt{3} + 3\sqrt[3]{2} - 6\sqrt{3} - 2\sqrt[3]{2} = \sqrt[3]{2}$
4	The left hand side $= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 2 \times 3\sqrt[3]{2} =$ zero = the right hand side.
5	$3\sqrt{3} + 3\sqrt[3]{2} - 3\sqrt{2} - \sqrt[3]{2} = 2\sqrt[3]{2}$
6	$2\sqrt[3]{2} - \sqrt[3]{2} - \sqrt[3]{2} = 0$
7	$\boxed{1}$ $(\sqrt[3]{5} + 1 - \sqrt[3]{5} + 1)^5 = 2^5 = 32$ $\boxed{2}$ $(\sqrt[3]{5} + 1 + \sqrt[3]{5} - 1)^3 = (2\sqrt[3]{5})^3 = 8 \times 5 = 40$
8	$\boxed{1}$ The left hand side $= 4\sqrt[3]{2} + 2\sqrt[3]{2} - 2 \times 3\sqrt[3]{2} =$ zero = the right hand side. $\boxed{2}$ The left hand side $= 3\sqrt[3]{2} \times 2\sqrt[3]{2} \div (6\sqrt[3]{4})$ $= 6\sqrt[3]{4} \div 6\sqrt[3]{4} = 1 =$ the right hand side.
9	$\because x - y = 3 + \sqrt[3]{6} - 3 + \sqrt[3]{6} = 2\sqrt[3]{6}$ $, x + y = 3 + \sqrt[3]{6} + 3 - \sqrt[3]{6} = 6$ $\therefore (\frac{x-y}{x+y})^3 = (\frac{2\sqrt[3]{6}}{6})^3 = (\frac{\sqrt[3]{6}}{3})^3 = \frac{(\sqrt[3]{6})^3}{3^3} = \frac{6}{27} = \frac{2}{9}$
10	(1) $2\sqrt[3]{3} - 10\sqrt[3]{3} = -8\sqrt[3]{3}$ (2) $5 - 2\sqrt[3]{3}$
11	(1) $\frac{1}{2} \times 6\sqrt[3]{10 \times 100} = 3\sqrt[3]{1000} = 3 \times 10 = 30$ (2) $\sqrt[3]{\frac{2}{5} \times \frac{4}{25}} = \sqrt[3]{\frac{8}{125}} = \frac{2}{5}$ (3) $\frac{3}{2}$

Lesson 9

Ex.(1):complete

1	$120cm^2$	10	$6L^2$
2	441	11	54
3	$4L^2$	12	$6cm^3$
4	96	13	3
5	125	14	$\frac{4}{3}\pi r^3$
6	$8L^3$	15	1cm
7	6cm	16	36
8	10cm	17	1.5
9	36	18	0.75

Ex.(2):choose

1	C	11	A
2	D	12	A
3	C	13	A
4	C	14	B
5	B	15	D
6	B	16	B
7	D	17	A
8	C	18	C
9	A	19	B
10	C	20	D

Ex.(3):Answer the following

1	The edge length $=\sqrt{36 \div 4} = 3cm$ (1)T.A=$6L^2 = 6\times3^2 = 54cm^2$ (2)V=$L^3 = 3^3 = 27cm^3$
2	The edge length=$12\div 4$=3cm (1) V=$L^3 = 3^3 = 27cm^3$ (2) L.A=$4L^2 = 4\times3^2 = 36cm^2$
3	The edge length=$60\div 12$=5cm (1) V=$L^3 = 5^3 = 125cm^3$ (2) T.A=$6L^2 = 6\times5^2 = 150cm^2$
4	S=$\sqrt{720 \div 5} = 12cm$, T.A=S×4×h+2×S×S=12×4×5+2×12×12=$528cm^2$
5	The edge length $=\sqrt{294 \div 6} = 7cm$, V. of cube =$L^3 = 7^3$ =343cm3 , V. of cuboid =L×w×h= $7\sqrt{2} \times 5\sqrt{2} \times 5 = 350cm^3$, V. Of cuboid is a greater
6	$\because$ The area of the circle $= \pi r^2$ $\therefore 64\pi = \pi r^2$ $\therefore r^2 = 64$ $\therefore r = \sqrt{64} = 8$ cm. The circumference of the circle $= 2\pi r$ $= 2 \times 3.14 \times 8 \approx 50$ cm.
7	$\because$ The area of the circle $= 2 \times 12.32 = 24.64$ cm^2. $\therefore \pi r^2 = 24.64$ $\therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$ $\therefore r = \sqrt{7.84} = 2.8$ cm. $\therefore$ The perimeter of the figure $= \pi r + 2r$ $= \frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.
8	The area of the shaded part = the area of the great circle – the area of the small circle $= \pi r_1^2 - \pi r_2^2$ $= \pi \times 25 - \pi \times 9 = 16\pi$ cm^2.
9	r = $\sqrt{924 \div (\frac{22}{7} \times 6)} = 7cm$, $L.A = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 6 = 264cm^2$
10	r = $\sqrt{7636 \div (3.14 \times 24)} = 10cm$, $T.A = 2\pi r(h+r)$ $= 2 \times 3.14 \times 10(24+10) = 2135.2cm^2$

11	r = $\sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{\frac{99000}{7} \div (\frac{4}{3} \times \frac{22}{7})} = 15cm$
12	r = $\sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{\frac{32}{3}\pi \div \frac{4}{3}\pi} = 2cm\ , d = 2r = 2 \times 2 = 4cm$
13	r = $\sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{36\pi \div \frac{4}{3}\pi} = 3cm$
14	r = $\sqrt{V \div \pi h} = \sqrt{90\pi \div 10\pi} = 3cm$
15	V=$\pi r^2 h = 3.14 \times 3.5^2 \times 10 = 384.65cm^3$
16	V.of cuboid=77×24×21=38808cm^3 , V.of cuboid=V.of sphere = 38808 cm^3 r = $\sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{38808 \div (\frac{4}{3} \times \frac{22}{7})} = 21cm$
17	The volume of the cylinder = $\pi r^2 h$ $= \frac{22}{7} \times (14)^2 \times 20 = 12320$ cm^3. The total area of the cylinder = $2\pi rh + 2\pi r^2$ $= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992$ cm^2.
18	r = $\sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{36\pi \div \frac{4}{3}\pi} = 3cm$
19	$V = 8\pi \div \frac{3}{4} = \frac{32}{3}\pi cm^3\ , r = \sqrt[3]{V \div \frac{4}{3}\pi} = \sqrt[3]{\frac{32}{3}\pi \div \frac{4}{3}\pi} = 2cm$
20	$A = 4\pi r^2 = 4 \times \frac{22}{7} \times 7^2 = 616cm^2$
21	$L.A = 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 15 = 660cm^2$

Lesson 10

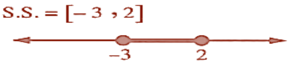
Ex.(1):complete

1	$[2,\infty[$	9	$\geq \frac{-3}{2}$
2	$\{\sqrt{2}\}$	10	$\leq 2\sqrt{2}$
3	$3-2\sqrt{2}$	11	$]2,4[$
4	$\{\sqrt{2}\}$	12	$]-2,5]$
5	$[1,\infty[$	13	$]2,\infty[$
6	≥ 3	14	6
7	< 3	15	$]-2,\infty[$
8	< -3	16	$[1,\infty[$

Ex.(2):choose

1	D	11	A
2	D	12	D
3	A	13	C
4	C	14	A
5	D	15	D
6	A	16	D
7	C	17	C
8	B	18	B
9	D	19	C
10	C		

Ex.(3):Answer the following

1	(1)5x=-5 , x=-1 , the S.S={-1} (2)1<x≤4 , the S.S=]1,4]
2	3<3X≤9 (÷3) 1<X≤3 , the S.S=]1,3] 
3	$\because 5x-2x<9+3$ $\therefore 3x<12$ $\therefore x<4$ $\therefore$ The S.S. $=]-\infty, 4[$ Represent by yourself the S.S. on the number line
4	$\because 3<2x-1<5$ $\therefore 4<2x<6$ $\therefore 2<x<3$ $\therefore$ The S.S. $=]2, 3[$ 
5	$\because 4x-4x\leq 5x+2-4x<4x+3-4x$ $\therefore 0\leq x+2<3$ $\therefore -2\leq x<1$ $\therefore$ The S.S. $=[-2, 1[$
6	$\because -x+2x\geq -3-1$ $\therefore x\geq -4$ $\therefore$ The S.S. $=[-4, \infty[$
7	$\because x-1-x<3x-1-x\leq x+1-x$ $\therefore -1<2x-1\leq 1$ $\therefore 0<2x\leq 2$ $\therefore 0<x\leq 1$ $\therefore$ The S.S. $=]0, 1]$ Represent by yourself the S.S. on the number line
8	$\because 2+2x-2x\leq 3x+3-2x<5+2x-2x$ $\therefore 2\leq x+3<5$ $\therefore -1\leq x<2$ $\therefore$ The S.S. $=[-1, 2[$ Represent by yourself the S.S. on the number line
9	$\because -4<-x\leq -2$ $\therefore 4>x\geq 2$ $\therefore$ The S.S. $=[2, 4[$ 
10	$\because -2\leq x+1\leq 3$ $\therefore -3\leq x\leq 2$ $\therefore$ The S.S. $=[-3, 2]$ 

11	$\because x+3-x \geq 2x-x \geq x-2-x$ $\therefore 3 \geq x \geq -2$ $\therefore$ The S.S. $= [-2, 3]$ Represent by yourself the S.S. on the number line
12	By multiplying by 6 $\therefore 3x-4 < 6x+6 < 3x+9$ $\therefore -4 < 3x+6 < 9$ $\therefore -10 < 3x < 3$ $\therefore -\frac{10}{3} < x < 1$ $\therefore$ The S.S. $=]-\frac{10}{3}, 1[$ Represent by yourself the S.S. on the number line

Unit 2

Lessons 1,2,3

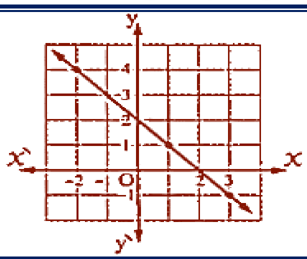
Ex.(1):choose

1	D	9	B	17	D	25	B
2	B	10	B	18	B	26	B
3	A	11	C	19	A	27	A
4	C	12	A	20	C	28	B
5	B	13	B	21	A	29	D
6	D	14	B	22	B	30	C
7	A	15	C	23	A	31	B
8	A	16	C	24	B	32	C

Ex.(2):Answer the following

1

x	1	3	-2
y	1	-1	4



2 Graph by your self

3 Graph by your self

4 $\because$ The straight line intersects x-axis at $(3, b)$

$\therefore b = 0$

$\therefore (3, 0)$ satisfies the relation $2x - y = a$

$\therefore 2 \times 3 - 0 = a$ $\therefore a = 6$

5 $\because (-3, 2)$ satisfies the relation $3x + by = 1$

$\therefore 3 \times (-3) + b \times 2 = 1$ $\therefore 2b = 9 + 1$

$\therefore 2b = 10$ $\therefore b = 5$

6

$\because$ (k , 2 k) satisfies the relation $x + y = 15$

$\therefore k + 2k = 15 \qquad \therefore 3k = 15 \qquad \therefore k = 5$

7

$\because$ (3 , a) satisfies the relation $y - 2x = 4$

$\therefore a - 2 \times 3 = 4 \qquad \therefore a = 10$

8

1

x	0	1	2	3
y	1	5	9	13

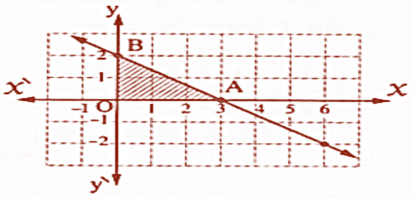
2

x	− 4	− 3	− 2
y	− 5	0	5

9

13 $y = -\frac{2}{3}x + 2$

x	0	3	6
y	2	0	− 2



From the graph :

The area of Δ OAB $= \frac{1}{2} \times 3 \times 2 = 3$ square units.

10

$\because$ The rate of consumption of fuel

$= \frac{\text{the amount of consumpted fuel}}{\text{time}}$

$= \frac{2.47}{3} = \frac{247}{300}$ litre/hr.

$\therefore$ the consumpted amount =

The rate of consumption $\times$ time

$= \frac{247}{300} \times 10 = 8\frac{7}{30}$ litre

11

$\because$ The uniform velocity $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{180}{3} = 60$ km./hr.

$\therefore$ the covered distance = the taken time $\times$ the

uniform velocity $= 60 \times 5 = 300$ km.

12

1 The velocity within the first 3 hours = the slope

of the striaght line $= \frac{60 - 20}{3 - 0} = \frac{40}{3} = 13\frac{1}{3}$ km./hr.

2 The velocity within the next 4 hours = the slope

of the straight line $= \frac{0 - 60}{7 - 3} = \frac{-60}{4} = -15$ km./hr.

The negative sign means that the bicycle returns back with velocity 15 km./hr.

The total distance = 40 + 60 = 100 km.

13

1 $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{2-1} = 1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-3-2}{-3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{7-(-3)}{-6-4} = \frac{10}{-10} = -1$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-4-7}{5-(-6)} = \frac{-11}{11} = -1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

3 $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-12}{2-(-2)} = \frac{-8}{4} = -2$

$\because$ The slope of $\overleftrightarrow{BC} = \frac{-4-4}{6-2} = \frac{-8}{4} = -2$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is a common point.

$\therefore$ The points A , B and C are collinear.

Unit 3

Lessons 1,2,3

Ex.(1):choose

1	B	9	D	17	C
2	A	10	B	18	A
3	B	11	C	19	C
4	C	12	C		
5	C	13	D		
6	A	14	B		
7	D	15	C		
8	A	16	D		

Ex.(2):Answer the following

1

Sets	Centre of Sets "x"	Frequency "f"	Center of sets × frequency "$x \times f$"
5 –	10	6	60
15 –	20	8	160
25 –	30	4	120
35 –	40	2	80
Total		20	420

$\therefore$ The mean $= \frac{420}{20} = 21$

2

Sets	"x"	"f"	"$x \times f$"
16 –	18	10	180
20 –	22	15	330
24 –	26	22	572
28 –	30	25	750
32 –	34	20	680
36 –	38	8	304
Total		100	2816

$\therefore$ The mean $= \frac{2816}{100} = 28.16$

3

Sets	"x"	"f"	"$x \times f$"
15 –	20	2	40
25 –	30	3	90
35 –	40	5	200
45 –	50	8	400
55 –	60	6	360
65 –	70	4	280
75 –	80	2	160
Total		30	1530

$\therefore$ The mean $= \frac{1530}{30} = 51$

4

Sets	"x"	"f"	"$x \times f$"
140 –	142	12	1704
144 –	146	20	2920
148 –	150	38	5700
152 –	154	22	3388
156 –	158	17	2686
160 –	162	11	1782
Total		120	18180

$\therefore$ The mean $= \frac{18180}{120} = 151.5$ cm.

5

Sets	"x"	"f"	"$x \times f$"
10 –	15	1	15
20 –	25	2	50
30 –	35	4	140
40 –	45	2	90
50 –	55	1	55
Total		10	350

$\therefore$ The mean of marks of students $= \frac{350}{10} = 35$ marks.

2 The number of failed students = 3 students.

6

1 $k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$

$\therefore k - 2 = 20 \qquad \therefore k = 22$

2

Sets	"x"	"f"	"$x \times f$"
2 –	4	4	16
6 –	8	5	40
10 –	12	8	96
14 –	16	20	320
18 –	20	7	140
22 –	24	5	120
26 –	28	1	28
Total		50	760

$\therefore$ The mean $= \frac{760}{50} = 15.2$ days.

7

$\because$ The total of marks of the student in 5 months

$= 5 \times 23.8 = 119$ marks.

, let the required mark of the sixth month be x

$\therefore \frac{119 + x}{6} = 24 \qquad \therefore 119 + x = 144$

$\therefore x = 144 - 119 = 25$ marks.

$\therefore$ The mark of the student in the 6^{th} month is 25

Lessons 4

Ex.(1):choose

1	C	9	A
2	A	10	B
3	C	11	B
4	B	12	B
5	A	13	C
6	A	14	B
7	B	15	C
8	B	16	A

Ex.(2):Answer the following

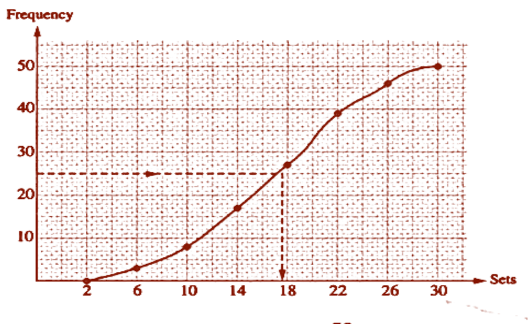
1

Sets	"x"	"f"	"$x \times f$"
2 –	4	3	12
6 –	8	5	40
10 –	12	9	108
14 –	16	10	160
18 –	20	12	240
22 –	24	7	168
26 –	28	4	112
Total		50	840

$\therefore$ The mean $= \frac{840}{50} = 16.8$

2 We form the ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median ≈ 17.6

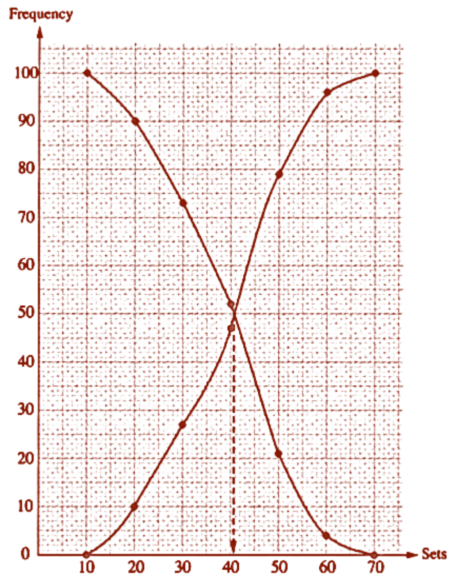
2

1 $x = 30$, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$

$\therefore k + 2 = 17$ $\quad \therefore k = 15$

2

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	100
less than 20	10	20 and more	90
less than 30	27	30 and more	73
less than 40	47	40 and more	53
less than 50	79	50 and more	21
less than 60	96	60 and more	4
less than 70	100	70 and more	0

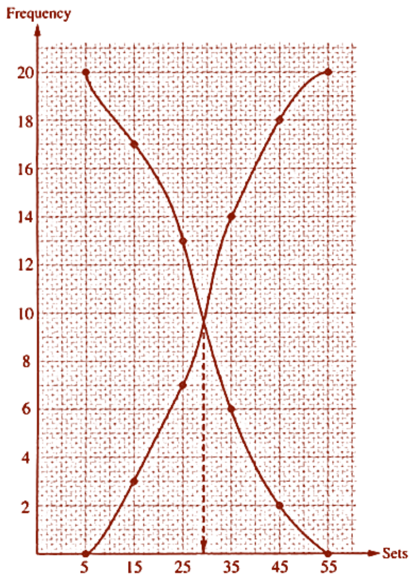


The median ≈ 41

3

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 15	3
less than 25	7
less than 35	14
less than 45	18
less than 55	20

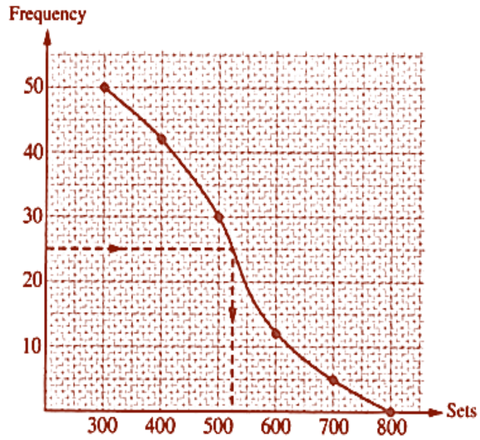
The lower limits of sets	Descending cumulative frequency
5 and more	20
15 and more	17
25 and more	13
35 and more	6
45 and more	2
55 and more	0



From the graph we find that the median ≈ 29 kg.

4

The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0



The order of the median = $\frac{50}{2} = 25$

The median wage = 520 pounds.

Lesson 5

Ex.(1):complete

1	The mode	11	46
2	The mode	12	4
3	The most common value	13	6
4	2	14	3
5	7	15	6
6	8	16	7
7	13	17	6
8	11	18	8
9	11	19	3
10	11	20	2

Ex.(1):choose

1	A
2	B
3	A
4	D
5	D
6	D
7	B
8	D

Ex.(2):Answer the following

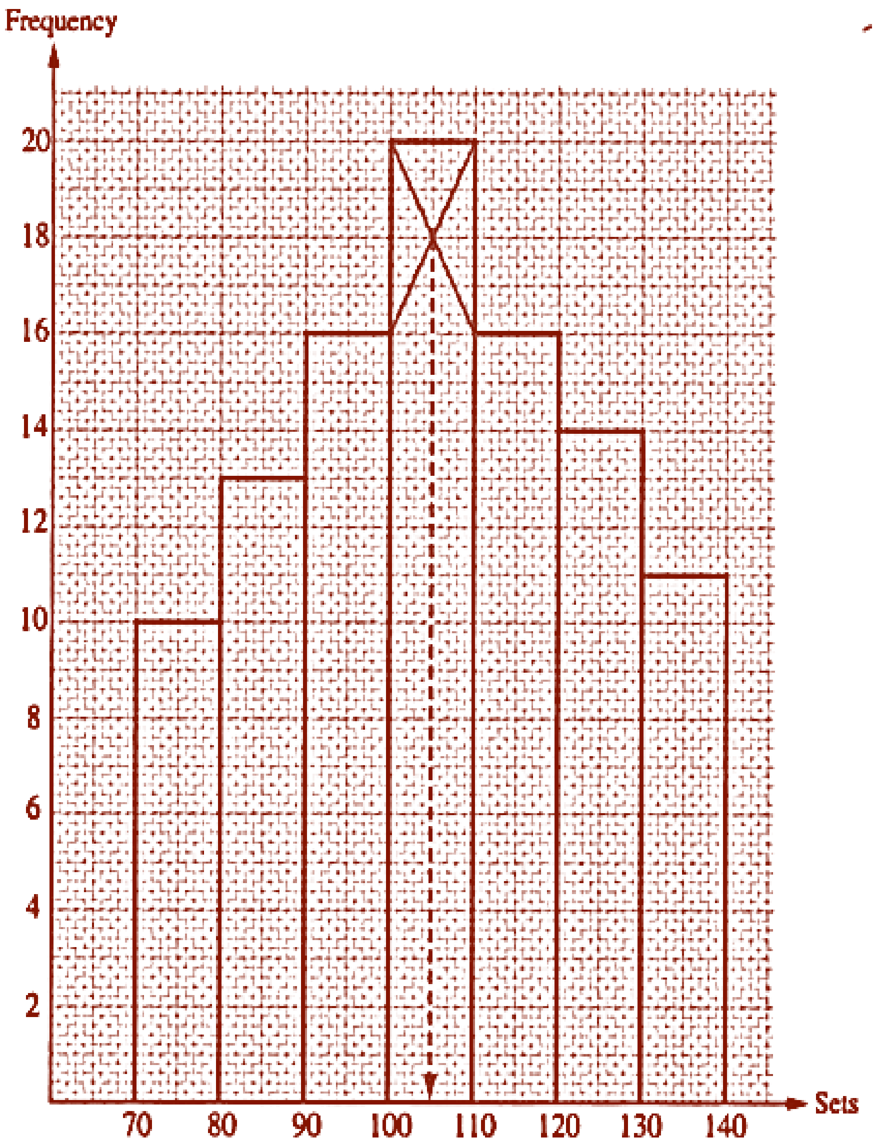
1

$\boxed{1}$ $x = 110$

, $k - 4 = 100 - (10 + 13 + 20 + 16 + 14 + 11)$

$\therefore k - 4 = 16 \qquad \therefore k = 20$

$\boxed{2}$



From the graph : The mode wage = 105 pounds.

2

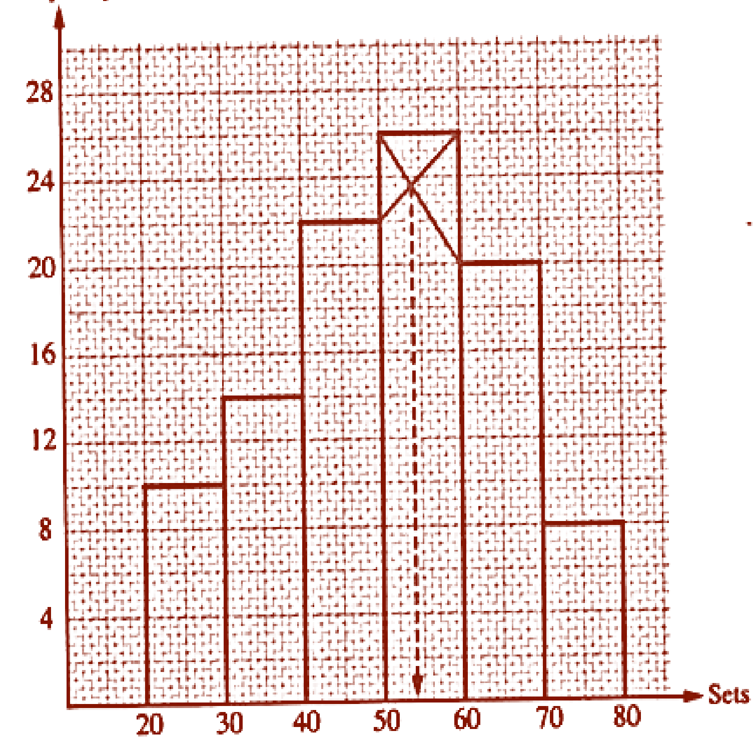
1 $k = 100 - (10 + 22 + 26 + 20 + 8) = 14$

2

Sets	"x"	"f"	"$x \times f$"
20 –	25	10	250
30 –	35	14	490
40 –	45	22	990
50 –	55	26	1430
60 –	65	20	1300
70 –	75	8	600
Total		100	5060

$\therefore$ The mean $= \frac{5060}{100} = 50.6$ pounds.

3



From the graph : The mode value = 54 pounds.

3

[1] $\because$ 3 k + 4 k = 50 − (7 + 10 + 8 + 4)

$\therefore$ 7 k = 21 $\therefore$ k = $\frac{21}{7}$ = 3

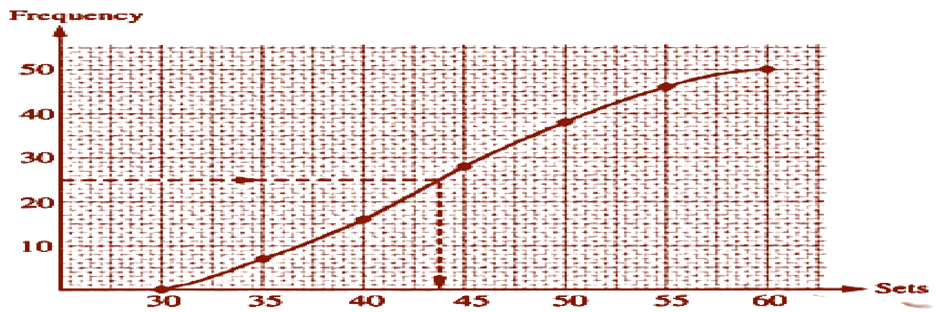
[2]

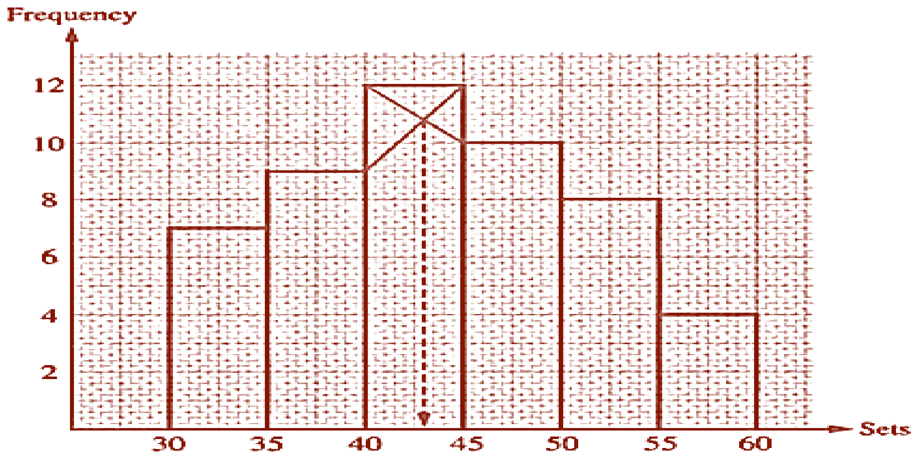
Sets	"x"	"f"	"$x \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

$\therefore$ The mean = $\frac{2200}{50}$ = 44 kg.

[3]

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50





From the graph : The mode weight = 43 kg.

[5] The order of the median = $\frac{50}{2}$ = 25

$\therefore$ The median $\approx$ 43.5 kg.

Guide Answers

Geometry

Unit 4

Lesson 1

Ex.(1):complete

1	The median
2	concurrent
3	One point
4	2:1
5	1:2
6	The base
7	concurrence
8	9cm
9	$\frac{2}{3}$
10	6cm
11	4

Ex.(2):Choose

1	A
2	D
3	A
4	C
5	A
6	B
7	A
8	C
9	D
10	D
11	C

Ex.(3):Answer the following

1

$\because$ $\overline{AD}$, $\overline{BE}$ are two medians in ΔABC ,

$\overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of ΔABC

$\therefore MD = \frac{1}{3} AD = \frac{1}{3} \times 6 = 2$ cm. (1)

, $ME = \frac{1}{3} BE = \frac{1}{3} \times 9 = 3$ cm. (2)

$\because$ D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ in ΔABC

$\therefore DE = \frac{1}{2} AB = \frac{1}{2} \times 9 = 4.5$ cm. (3)

From (1) , (2) and (3) :

$\therefore$ The perimeter of $\Delta MDE = 2 + 3 + 4.5 = 9.5$ cm.

(The req.)

2

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\,DE$ $\therefore BC = 8$ cm.

$\because$ M is the intersection point of medians of ΔABC

$\therefore MC = 2\,DM$ $\therefore MC = 6$ cm.

, $BM = \frac{2}{3} BE$ $\therefore BM = 4$ cm.

$\therefore$ The perimeter of $\Delta BMC = 8 + 6 + 4 = 18$ cm.

(The req.)

3

$\because$ M is the intersection point of the medians of ΔABC

$\therefore XM = \frac{1}{2} MC = 4$ cm.

$\therefore$ The perimeter of $\Delta MXY = 4 + 5 + 3 = 12$ cm.

(First req.)

, $AM = 2\,MY = 6$ cm.

$\because$ X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{BC}$

$\therefore AC = 2\,XY = 10$ cm.

$\therefore$ The perimeter of $\Delta MAC = 6 + 8 + 10 = 24$ cm.

(Second req.)

4

$\because$ M is the intersection point of medians of ΔABC

$\therefore MF = \frac{1}{2} AM$ (1)

, $MD = \frac{1}{2} MC$ (2)

$\because$ D is the midpoint of $\overline{AB}$,

F is the midpoint of $\overline{BC}$ in ΔABC

$\therefore DF = \frac{1}{2} AC$ (3)

By adding (1) , (2) and (3) :

$\therefore MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$

$\therefore$ The perimeter of ΔMFD

$= \frac{1}{2} (AM + MC + AC)$

$= \frac{1}{2}$ the perimeter of ΔAMC

$= \frac{1}{2} \times 36 = 18$ cm. (The req.)

5	$\because$ M is the point of concurrence of the medians of ΔABC $\therefore \overline{CD}$ is a median in ΔABC $\therefore DM = \frac{1}{2} MC = 3$ cm. , $\because \Delta AMD$ is a right-angled triangle at M $\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$ $\therefore AM = 4$ cm. $\therefore ME = \frac{1}{2} AM = 2$ cm. (The req.)	6	DF = 9cm DE = 8cm FE = 10cm Perimeter = 9+8+10 = 27 cm
7	$\because$ M is the point of concurrence of the medians of ΔABC $\therefore \overline{CD}$ is a median in ΔABC $\therefore$ D is the midpoint of $\overline{AB}$ In ΔAMB : $\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BM}$ $\therefore \overline{MD}$, $\overline{AE}$ are two medians in ΔAMB $\therefore$ N is the point of concurrence of the medians of ΔAMB $\therefore MN = 2\ ND$ $\therefore x + 3 = 2(x - 1)$ $\therefore x + 3 = 2x - 2$ $\therefore 3 + 2 = 2x - x$ $\therefore x = 5$ $\therefore ND = 5 - 1 = 4$ cm. , $MN = 5 + 3 = 8$ cm. $\therefore MD = ND + MN = 12$ cm. , $\because \overline{CD}$ is a median in ΔABC $\therefore MC = 2\ MD = 24$ cm. (The req.)		

Lesson 2

Ex.(1):complete

1	half	8	5
2	Half the length of hypotenuse	9	4
3	The angle of this vertex is right	10	$\frac{1}{2}$
4	half	11	30°
5	Half the length of hypotenuse	12	60°
6	twice	13	60
7	8	14	21

Ex.(2):Choose

1	B	8	B
2	C	9	B
3	B	10	A
4	A	11	B
5	D	12	D
6	A	13	D
7	B	14	B

Ex.(3):Answer the following

1

In Δ ADC :

$\because m(\angle D) = 90^\circ$, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} AC$ (1)

In Δ ABC :

$\because m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$

$\therefore AB = \frac{1}{2} AC$ (2)

From (1) and (2) :

$\therefore AB = DE$ (Q.E.D.)

2

In Δ LXZ :

$\because$ D is the midpoint of $\overline{LX}$, E is the midpoint of $\overline{LZ}$

$\therefore DE = \frac{1}{2} XZ$ (1)

From Δ XYZ :

$\because m(\angle Y) = 90^\circ$, M is the midpoint of $\overline{XZ}$

$\therefore YM = \frac{1}{2} XZ$ (2)

From (1) and (2) :

$\therefore DE = YM$ (Q.E.D.)

3

In Δ ACD :

$\because$ E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$

$\therefore EF = \frac{1}{2} AC$ $\therefore AC = 8$ cm.

In Δ ABC :

$\because m(\angle B) = 90^\circ$, $m(\angle ACB) = 30^\circ$

$\therefore AB = \frac{1}{2} AC = 4$ cm. (The req.)

4

$\because \angle ADC$ is an exterior angle of Δ ABD

$\therefore m(\angle ADC) = 33^\circ + 27^\circ = 60^\circ$

$\therefore$ In Δ ADC :

$m(\angle DAC) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

$\therefore DC = \frac{1}{2} AD$

$\therefore AD = 8$ cm. (The req.)

5	In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30° $\therefore$ AB = $\frac{1}{2}$ AC , $\because$ AB = DE = 5 cm. $\therefore$ DE = $\frac{1}{2}$ AC $\because$ $\overline{DE}$ is a median in Δ ACD $\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)	6	In Δ DBC : $\because$ E is the midpoint of $\overline{BC}$, $\overline{EF}$ // $\overline{BD}$ $\therefore$ EF = $\frac{1}{2}$ BD , $\because$ AM = EF $\therefore$ AM = $\frac{1}{2}$ BD $\because$ $\overline{AM}$ is a median in Δ ABD $\therefore$ m ($\angle$ BAD) = 90° (Q.E.D.)

Lesson 3

Ex.(1):complete

1	congruent	5	C , 50
2	40°	6	equilateral
3	70°	7	45
4	45	8	5

Ex.(2):Choose

1	A	11	D
2	A	12	C
3	B	13	B
4	B	14	C
5	D	15	D
6	D	16	B
7	C	17	D
8	B	18	B
9	B	19	D
10	C		

Ex.(3):Answer the following

1

In ΔABC :

$\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$= \frac{180^\circ - 40^\circ}{2} = 70^\circ$ (First req.)

$\because m(\angle ABC) = m(\angle ACB)$,

$\angle ABD$ supplements $\angle ABC$

, $\angle ACE$ supplements $\angle ACB$

$\because$ The supplementaries of the congruent angles are congruent

$\therefore \angle ABD \equiv \angle ACE$ (Second req.)

2

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle ACB) = 70^\circ$

$\therefore m(\angle BAC) = 180^\circ - (2 \times 70^\circ) = 40^\circ$

In ΔACD :

$\because AC = CD$ $\therefore m(\angle CAD) = m(\angle D)$

$\because \angle ACB$ is an exterior angle of ΔACD

$\therefore m(\angle ACB) = m(\angle CAD) + m(\angle D)$

$\therefore m(\angle CAD) = \frac{70^\circ}{2} = 35^\circ$

$\therefore m(\angle BAD) = m(\angle BAC) + m(\angle CAD)$

$= 40^\circ + 35^\circ = 75^\circ$ (The req.)

3

$\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ACD) = 30^\circ + 40^\circ = 70^\circ$

From ΔACD :

$\because AC = AD$

$\therefore m(\angle D) = m(\angle ACD) = 70^\circ$ (First req.)

$\therefore m(\angle CAD) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$ (Second req.)

4

$\because \Delta ACD$ is an equilateral triangle

$\therefore m(\angle CAD) = 60^\circ$ (1)

From ΔABC :

$\because AB = BC$

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle BAD) = 60^\circ + 70^\circ = 130^\circ$ (The req.)

5

$\because \Delta ACD$ is an equilateral triangle

$\therefore m(\angle CAD) = 60^\circ$ (1)

From ΔABC :

$\because AB = BC$

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle BAD) = 60^\circ + 70^\circ = 130^\circ$ (The req.)

6

$\because \angle LZX$ is an exterior angle of ΔXYZ

$\therefore m(\angle X) + m(\angle Y) = 130^\circ$

$\because ZX = ZY$ $\therefore m(\angle X) = m(\angle Y)$

$\therefore m(\angle Y) = \frac{130^\circ}{2} = 65^\circ$

$\because \overrightarrow{LM} \parallel \overline{XY}$, $\overleftrightarrow{LY}$ is a transversal to them

$\therefore m(\angle MLY) = m(\angle Y) = 65^\circ$ (The req.)

7

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore 2X + 13 = 3X - 17$ $\therefore X = 30^\circ$

$\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73^\circ$

$\therefore m(\angle A) = 180^\circ - (73^\circ + 73^\circ) = 34^\circ$ (The req.)

Lesson 4

Ex.(1):complete

1	equilateral	5	C , 50
2	equilateral	6	equilateral
3	120	7	45
4	Measure of its non-adjacent interior angle		

Ex.(2):Choose

1	C
2	B
3	B
4	A

Ex.(3):Answer the following

1

In Δ ADE : $\because \angle ADE \equiv \angle AED$ $\quad \therefore$ AD = AE

$D \in \overline{BC}$, $E \in \overline{BC}$

$\because \angle$ ADB supplements $\angle$ ADE ,

$\angle$ AEC supplements $\angle$ AED

but m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC)

(supplementaries of the congruent angles are congruent)

$\therefore$ $\Delta\Delta$ ADB , AEC in them :

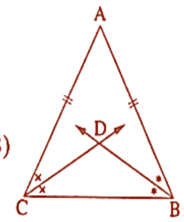
$\begin{cases} m(\angle ADB) = m(\angle AEC) \\ AD = AE \\ BD = CE \end{cases}$

$\therefore \Delta ADB \equiv \Delta AEC$

We deduce that AB = AC

$\therefore$ Δ ABC is an isosceles triangle. (Q.E.D.)

2



$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore \frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

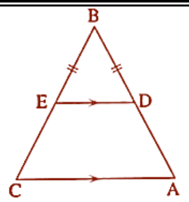
$\because$ m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC)

m ($\angle$ DCB) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) $\quad \therefore$ DB = DC

$\therefore$ Δ DBC is an isosceles triangle. (Q.E.D.)

3



From Δ EBD : $\because$ DB = EB

$\therefore$ m ($\angle$ BDE) = m ($\angle$ BED) (1)

$\because \overline{DE} \parallel \overline{AC}$, $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ m ($\angle$ A) = m ($\angle$ BDE)

(corresponding angles) (2)

Similarly m ($\angle$ C) = m ($\angle$ BED) (3)

From (1) , (2) and (3) : $\quad \therefore$ m ($\angle$ A) = m ($\angle$ C)

$\therefore$ AB = BC (Q.E.D.)

4

$\because$ Δ ABC is an equilateral triangle

$\therefore$ m ($\angle$ ACB) = 60°

$\because \angle$ ACB is an exterior angle of Δ DCF

$\therefore$ m ($\angle$ D) = 60° – 30° = 30°

$\therefore$ m ($\angle$ D) = m ($\angle$ F) $\quad \therefore$ CD = CF

$\therefore$ Δ DCF is an isosceles triangle. (Q.E.D.)

5

$\because$ m ($\angle$ ABC) = m ($\angle$ ACB) $\quad \therefore$ AB = AC

$\therefore$ $\Delta\Delta$ ADB , AEC in them :

$\begin{cases} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90^\circ \end{cases}$

$\therefore \Delta ADB \equiv \Delta AEC$

$\therefore$ m ($\angle$ DAB) = m ($\angle$ CAE) (Q.E.D.)

6

$\because$ AB = AC $\quad \therefore$ m ($\angle$ B) = m ($\angle$ C) (1)

$\because \overline{AB} \parallel \overline{DE}$, $\overleftrightarrow{BE}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ DEF) (corresponding angles) (2)

similarly m ($\angle$ C) = m ($\angle$ DFE) (3)

from (1) , (2) and (3)

$\therefore$ m ($\angle$ DEF) = m ($\angle$ DFE)

$\therefore$ DE = DF (Q.E.D. 1)

In $\Delta\Delta$ ABC , DEF :

$\because$ m ($\angle$ B) = m ($\angle$ DEF) , m ($\angle$ C) = m ($\angle$ DFE)

$\therefore$ m ($\angle$ BAC) = m ($\angle$ EDF) (Q.E.D. 2)

Lesson 5

Ex.(1):complete

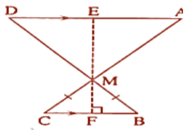
1	bisects the vertex and perpendicular to the base	9	3
2	the vertex , the base	10	0
3	bisects the base , perpendicular to it	11	$\triangle$ ABC
4	$\overline{yz}$, <x	12	CB
5	axis of symmetry	13	Perpendicular to the middle of it
6	3	14	Its terminals
7	1	15	AC
8	0	16	The straight line drawn from the vertex angle perpendicular to its base

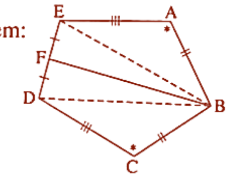
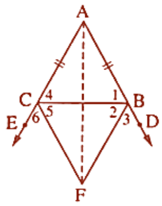
Ex.(2):Choose

1	D	8	C
2	B	9	A
3	C	10	B
4	B	11	D
5	A	12	A
6	C	13	B
7	C	14	A

Ex.(3):Answer the following

1	In $\triangle ABC$: $\because AB = AC$, $\overrightarrow{AE}$ bisects $\angle BAC$ $\therefore BE = \frac{1}{2} BC$ (Q.E.D.1) , $\overline{AE} \perp \overline{BC}$ $\therefore \overleftrightarrow{AE}$ is the axis of symmetry of $\overline{BC}$, $D \in \overleftrightarrow{AE}$ $\therefore BD = CD$ (Q.E.D.2)	2	In $\triangle ABC$: $\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BC = 2 \times 5 = 10$ cm. (First req.) In the right-angled triangle ADB at D $AD = \sqrt{(13)^2 - (5)^2} = 12$ cm. $\therefore$ The area of $\triangle ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm^2. (Second req.)

3	$\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them
	$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)
	but $m(\angle ABD) = m(\angle DBC)$
	$\therefore m(\angle ADB) = m(\angle ABD)$
	$\therefore$ In $\triangle ABD$: $AB = AD$ (Q.E.D.1)
	, $\because \overrightarrow{AE}$ bisects $\angle BAD$ $\therefore \overline{AE} \perp \overline{BD}$ (Q.E.D.2)
	, $BE = ED$ (Q.E.D.3)
		4	**Constr. :**
			Draw $\overleftrightarrow{MF} \perp \overline{BC}$ to meet $\overline{BC}$ at F and $\overline{AD}$ at E
			
			Proof : $\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.
			$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$
			$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$
			$\therefore m(\angle A) = m(\angle D)$ $\therefore AM = DM$
			$\therefore \triangle AMD$ is an isosceles triangle. (Q.E.D.1)
			In $\triangle MBC$: $\because MB = MC$, $\overline{MF} \perp \overline{BC}$
			$\therefore \overleftrightarrow{MF}$ is the axis of symmetry of $\triangle MBC$
			$\because \overline{AD} // \overline{BC}$, $\overleftrightarrow{FE}$ is a transversal to them.
			$\therefore m(\angle AEM) = m(\angle BFM) = 90^\circ$
			$\therefore \overline{ME} \perp \overline{AD}$
			, $\because MA = MD$ $\therefore \overleftrightarrow{ME}$ is the axis of $\triangle AMD$
			$\therefore \overleftrightarrow{EF}$ is the axis of symmetry of each of $\triangle AMD$, $\triangle BMC$ (Q.E.D.2)

<table>
<tr>
<td>5</td>
<td>In ΔXYL : ∵ $XL = XY$, M is the midpoint of $\overline{LY}$
∴ $\overleftrightarrow{XM}$ is the axis of $\overline{LY}$
similarly in ΔZYL , $\overleftrightarrow{ZM}$ is the axis of $\overline{LY}$
∴ X , M and Z are on the same straight line. (Q.E.D.)</td>
<td>6</td>
<td>Constr. : Draw $\overline{BD}$, $\overline{BE}$

Proof: ΔΔ ABE , CBD in them:
m (∠ A) = m (∠ C)
AB = CB ,
AE = CD
∴ Δ ABE ≡ Δ CBD
, then we deduce that: BE = BD
, ∵ $\overline{BF}$ is a median of Δ BED which is isosceles
∴ $\overline{BF} \perp \overline{DE}$ (Q.E.D.)</td>
</tr>
<tr>
<td>7</td>
<td>
∵ AB = AC
∴ m (∠ 1) = m (∠ 4) (1)
∵ m (∠ DBC) = 180° – m (∠ 1) (2)
, m (∠ BCE) = 180° – m (∠ 4) (3)
From (1) , (2) and (3) :
∴ m (∠ DBC) = m (∠ BCE)
∴ $\frac{1}{2}$ m (∠ DBC) = $\frac{1}{2}$ m (∠ BCE)
∴ m (∠ 2) = m (∠ 5) ∴ FB = FC
∴ Δ BFC is an isosceles triangle. (Q.E.D.1)
∵ AB = AC , FB = FC
∴ $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$ (Q.E.D.2)</td>
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</table>

Unit 5

Lessons 1,2

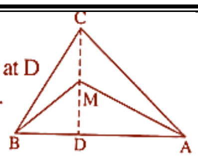
Ex.(1):complete

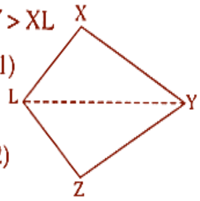
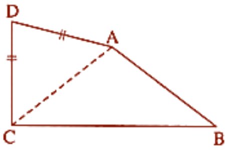
1	The angle of the greater measure
2	Greater measure
3	>
4	>
5	B

Ex.(2):Choose

1	B	5	B
2	B	6	B
3	B	7	A
4	D		

Ex.(3):Answer the following

1	$\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal. $\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles) $\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$ $\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1) $\because E \in \overrightarrow{CD}$ $\therefore \angle ADE$ is an exterior angle of ΔACD $\therefore m(\angle ADE) > m(\angle ACD)$ (2) From (1) and (2) : $\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)	2	 Const : Draw $\overrightarrow{CM}$ to intersect $\overline{BA}$ at D Proof : $\because \angle AMD$ is an exterior angle of ΔAMC $\therefore m(\angle AMD) > m(\angle ACM)$ (1) $\because \angle BMD$ is an exterior angle of ΔCMB $\therefore m(\angle BMD) > m(\angle BCM)$ (2) Adding (1) and (2) $\therefore m(\angle AMD) + m(\angle BMD) > m(\angle ACM) + m(\angle BCM)$ $\therefore m(\angle AMB) > m(\angle C)$ (Q.E.D.)
3	In ΔAXY : $\because m(\angle AXY) = m(\angle AYX)$ $\therefore AX = AY$ (1) $\because AC > AB$ $\therefore AY + YC > AX + XB$ (2) From (1) and (2) : $\therefore YC > XB$ (Q.E.D.)	4	1 $\because \overline{BC}$ is the longest side $\therefore \angle A$ is the greatest angle in measure $\because \overline{AC}$ is the shortest side $\therefore \angle B$ is the smallest angle in measure $\therefore$ The ascending order of measures of the angles is : $m(\angle B)$, $m(\angle C)$ and $m(\angle A)$ 2 $\because \overline{BC}$ is the longest side. $\therefore \angle A$ is the greatest angle in measure. $\because \overline{AB}$ is the shortest side. $\therefore \angle C$ is the smallest angle in measure. $\therefore$ The ascending order of the measures of the angles is : $m(\angle C)$, $m(\angle B)$ and $m(\angle A)$

5	In ΔABC : $\because AC > AB$ $\therefore m(\angle ABC) > m(\angle ACB)$ (1) In ΔBDC : $\because DB = DC$ $\therefore m(\angle DBC) = m(\angle DCB)$ (2) Adding (1) and (2) : $\therefore m(\angle ABC) + m(\angle DBC) > m(\angle ACB) + m(\angle DCB)$ $\therefore m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)	**6**	**Construction :** Draw $\overline{YL}$ **Proof :** In ΔXYL $\because XY > XL$  $\therefore m(\angle XLY) > m(\angle XYL)$ (1) In ΔZYL : $\because YZ > ZL$ $\therefore m(\angle ZLY) > m(\angle ZYL)$ (2) Adding (1) and (2) : $\therefore m(\angle XLY) + m(\angle ZLY) > m(\angle XYL) + m(\angle ZYL)$ $\therefore m(\angle XLZ) > m(\angle XYZ)$ (Q.E.D.)
7	 **Construction :** Draw $\overline{AC}$ **Proof :** In ΔABC $\because BC > AB$ $\therefore m(\angle BAC) > m(\angle ACB)$ (1) In ΔDAC : $\because DA = DC$ $\therefore m(\angle DAC) = m(\angle DCA)$ (2) Adding (1) and (2) : $\therefore m(\angle BAC) + m(\angle DAC) > m(\angle ACB) + m(\angle DCA)$ $\therefore m(\angle BAD) > m(\angle BCD)$ (Q.E.D.)	**8**	In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1) $\because \overline{XY} \parallel \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal. $\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles) (2) Similarly : $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal. $\therefore m(\angle AXY) = m(\angle B)$ (3) From (1) , (2) and (3) : $\therefore m(\angle AYX) > m(\angle AXY)$ (Q.E.D.)
9	$\because \Delta ABC$ is an equilateral triangle. $\therefore m(\angle ABC) = m(\angle ACB) = 60°$ $\because m(\angle EBC) < m(\angle ECB)$ Subtracting $\therefore m(\angle ABC) - m(\angle EBC) > m(\angle ACB) - m(\angle ECB)$ $\therefore m(\angle ABE) > m(\angle ACE)$ (1) (Q.E.D.1) $\because m(\angle A) = m(\angle B)$ $\therefore m(\angle A) = m(\angle ABE) + m(\angle EBC)$ $\therefore m(\angle A) > m(\angle ABE)$ and from (1) : $\therefore m(\angle A) > m(\angle ABE) > m(\angle ACE)$ (Q.E.D.2)	**10**	In ΔABM : $\because AM > BM$ $\therefore m(\angle ABM) > m(\angle A)$ (1) $\because AM = CM$ $\therefore$ In ΔCBM : $MC > MB$ $\therefore m(\angle MBC) > m(\angle C)$ (2) Adding (1) and (2) : $\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$ $\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$ $\therefore \angle ABC$ is an obtuse angle. (Q.E.D)

Lesson 3

Ex.(1):complete

1	hypotenuse	8	XY	15	BC	22	BC
2	hypotenuse	9	BC	16	AB	23	>
3	hypotenuse	10	AC	17	BC , AC	24	
4	Opposite to a side greater in length than that opposite to the other angle	11	BC	18	BC	25	
5	The smallest side in length	12	AB	19	1	26	
6	The greatest side in length	13	AC	20	AB		
7	>	14	DF	21	>		

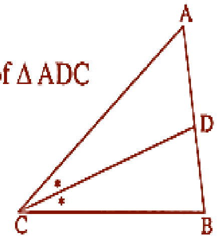
Ex.(2):Choose

1	A	8	A	15	C	22	A
2	A	9	C	16	A	23	C
3	B	10	A	17	D	24	C
4	C	11	A	18	C	25	B
5	A	12	B	19	B	26	C
6	B	13	B	20	A		
7	A	14	B	21	A		

Ex.(3):Answer the following

1	$\because C \in \overrightarrow{AE}$ $\therefore m(\angle ACB) = 180^\circ - 120^\circ = 60^\circ$ $\because B \in \overrightarrow{CD}$ $\therefore m(\angle ABC) = 180^\circ - 110^\circ = 70^\circ$ $\therefore m(\angle A) = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$ $\therefore m(\angle ACB) > m(\angle A)$ $\therefore AB > BC$ (Q.E.D.)	2	$\Delta ABC : \because AB = AC$ $\therefore m(\angle ACB) = m(\angle B) = 65^\circ$ $\therefore m(\angle DCB) = 65^\circ + 20^\circ = 85^\circ$ In ΔDBC : $\therefore m(\angle D) = 180^\circ - (65^\circ + 85^\circ) = 30^\circ$ $\therefore$ In ΔDAC : $m(\angle D) > m(\angle ACD)$ $\therefore AC > AD$ but $AB = AC$ $\therefore AB > AD$ (Q.E.D.)

3



$\because \angle BDC$ is an exterior angle of ΔADC

$\therefore m(\angle BDC) > m(\angle ACD)$

$\because m(\angle BCD) = m(\angle ACD)$

$\therefore m(\angle BDC) > m(\angle BCD)$

In ΔDBC : $\therefore BC > BD$ (Q.E.D.)

4

In ΔDBC : $\because DB = DC$

$\therefore m(\angle B) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$

$\because \overrightarrow{CD}$ bisects $\angle ACB$ $\therefore m(\angle ACD) = 40°$

$\because D \in \overline{AB}$

$\therefore m(\angle ADC) = 180° - 100° = 80°$

$\therefore$ In ΔADC : $m(\angle A) = 180° - (40° + 80°) = 60°$

$\therefore m(\angle ADC) > m(\angle A)$

$\therefore AC > DC$ but $DC = DB$

$\therefore AC > DB$ (Q.E.D.)

5

$\because \overleftrightarrow{AD} \mathbin{//} \overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle ACB) = m(\angle DAC) = 30°$ (alternate angles)

In ΔABC : $\because m(\angle BAC) > m(\angle ACB)$

$\therefore BC > AB$ (Q.E.D.)

6

In ΔABC : $\because AB > AC$

$\therefore m(\angle ABC) < m(\angle ACB)$

, $\because B \in \overrightarrow{AD}$, $C \in \overrightarrow{AE}$

$\therefore 180° - m(\angle ABC) > 180° - m(\angle ACB)$

$\therefore m(\angle CBD) > m(\angle BCE)$

$\because \overrightarrow{BF}$ bisects $\angle DBC$, $\overrightarrow{CF}$ bisects $\angle BCE$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)

$\therefore CF > BF$ (Q.E.D.2)

7

In ΔABD : $\because BD = AD$

$\therefore m(\angle BAD) = m(\angle B)$

$\therefore m(\angle BAD) + m(\angle DAC) > m(\angle B)$

$\therefore m(\angle BAC) > m(\angle B)$ $\therefore BC > AC$ (Q.E.D.)

8

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180°$

$\therefore 5x + 2° + 6x - 10° + x + 20° = 180°$

$\therefore 12x + 12° = 180°$ $\therefore 12x = 180° - 12° = 168°$

$\therefore x = \frac{168°}{12°} = 14°$ $\therefore m(\angle A) = 5 \times 14° + 2° = 72°$

, $m(\angle B) = 6 \times 14° - 10° = 74°$

$m(\angle C) = 14° + 20° = 34°$

$\therefore AB < BC < AC$ (The req.)

9

$AC > AB > BC$

10

In ΔABC : $\because \angle B$ is an obtuse angle.

$\therefore m(\angle B) > m(\angle C)$ (1)

$\because \overline{DE} \mathbin{//} \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal.

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles) (2)

$\because \overline{DE} \mathbin{//} \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal.

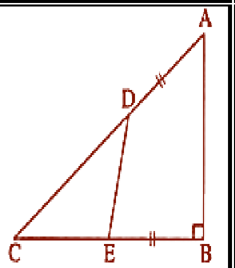
$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle ADE) > m(\angle AED)$

$\therefore AE > AD$ (Q.E.D.)

11



In ΔABC : $\because m(\angle B) = 90°$

$\therefore AC > BC$

$\because AD = BE$

$\therefore AC - AD > BC - BE$

$\therefore DC > EC$

$\therefore$ In ΔDEC : $\because DC > EC$

$\therefore m(\angle CED) > m(\angle CDE)$ (Q.E.D.)

12

In ΔACM : $\because m(\angle C) = 90°$ $\therefore AM > CM$ (1)

In ΔBDM : $\because m(\angle D) = 90°$ $\therefore BM > DM$ (2)

Adding (1) and (2) : $\therefore AM + MB > CM + MD$

$\therefore AB > CD$ (Q.E.D.)

Lesson 4

Ex.(1):complete

1	Greater than	8	6 , 12
2	AC	9	10cm
3	Is less than	10	7
4	2 , 8	11	8cm
5	1 , 7	12	6
6	1 , 9	13	8
7	1 , 15	14	4

Ex.(2):Choose

1	B	8	A	15	C	22	D
2	C	9	B	16	D	23	C
3	D	10	A	17	C	24	A
4	B	11	C	18	B	25	A
5	B	12	D	19	C	26	C
6	C	13	B	20	C		
7	C	14	A	21	B		

Ex.(3):Answer the following

1

From Δ ABM : MA + MB > AB

(Triangle inequality) (1)

From Δ BMC : MB + MC > BC

(Triangle inequality) (2)

From Δ AMC : MA + MC > AC (Triangle inequality) (3)

2

Assuming that ABC is a triangle

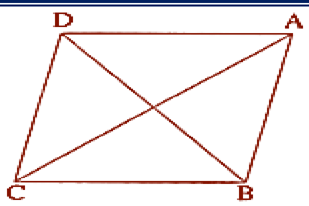
$\because$ AB < AC + BC adding AB to both sides

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

3



Let ABCD be a quadrilateral

In Δ ABC : AB + BC > AC (1)

In Δ BCD : BC + CD > BD (2)

In Δ ACD : AD + CD > AC (3)

In Δ ABD : AB + AD > BD (4)

Adding (1) , (2) , (3) , and (4) :

$\because$ 2 AB + 2 BC + 2 CD + 2 AD > 2 AC + 2 BD

$\therefore$ AB + BC + CD + AD > AC + BD

$\therefore$ The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral. (Q.E.D.)